Differentiability and the tangent plane

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Practice problems

1. Find the equation of the tangent plane to the surface $z = x^3 + y^3$ at the point $(1, 2)$.
2. Determine if the function $f(x, y) = |x| + |y|$ is differentiable at the point $(0, 0)$.
3. Find the tangent plane to $z = e^{xy}$ at the point $(0, 1)$.

Solutions

1. Find the equation of the tangent plane to the surface $z = x^3 + y^3$ at the point $(1, 2)$.

   - Step 1: Find $z_0 = 1^3 + 2^3 = 1 + 8 = 9$.
   - Step 2: Compute partial derivatives: $f_x = 3x^2$ and $f_y = 3y^2$.
   - Step 3: Evaluate partials at $(1, 2)$: $f_x(1, 2) = 3(1)^2 = 3$ and $f_y(1, 2) = 3(2)^2 = 12$.
   - Step 4: Plug into the formula: $z - 9 = 3(x - 1) + 12(y - 2)$.
   - Step 5: Simplify: $z = 3x - 3 + 12y - 24 + 9 = 3x + 12y - 18$.

   Answer: $z = 3x + 12y - 18$

2. Determine if the function $f(x, y) = |x| + |y|$ is differentiable at the point $(0, 0)$.

   - Step 1: Check partial derivatives at $(0, 0)$. $f_x(0, 0) = \lim_{h\to 0} \frac{|h|+0-0}{h} = \lim_{h\to 0} \frac{|h|}{h}$.
   - Step 2: The limit $\lim_{h\to 0} \frac{|h|}{h}$ does not exist because it is 1 from the right and -1 from the left.
   - Step 3: Since the partial derivatives do not exist at $(0, 0)$, the function cannot be differentiable there.

   Answer: Not differentiable

3. Find the tangent plane to $z = e^{xy}$ at the point $(0, 1)$.

   - Step 1: Find $z_0 = e^{0\cdot 1} = e^0 = 1$.
   - Step 2: Compute partials: $f_x = ye^{xy}$ and $f_y = xe^{xy}$.
   - Step 3: Evaluate at $(0, 1)$: $f_x(0, 1) = 1 \cdot e^0 = 1$ and $f_y(0, 1) = 0 \cdot e^0 = 0$.
   - Step 4: Plug into formula: $z - 1 = 1(x - 0) + 0(y - 1)$.
   - Step 5: Simplify: $z = x + 1$.

   Answer: $z = x + 1$
