Partial derivatives — deepened

Lesson 10 · Module 1 — Differentiation in Several Variables · Multivariable & Vector Calculus · dailymaths.uk/vectorcalc

Practice problems

1. Find all second order partial derivatives f_{xx}, f_{yy}, f_{xy}, and f_{yx} for the function $f(x, y) = 3x^3y^2 - 5x^2y + 7$.
2. Given $f(x, y) = e^{xy}$, show that $f_{xy} = f_{yx}$.
3. Find the mixed partial derivative f_{xy} for $f(x, y) = \sec(x + y)$.

Solutions

1. Find all second order partial derivatives f_{xx}, f_{yy}, f_{xy}, and f_{yx} for the function $f(x, y) = 3x^3y^2 - 5x^2y + 7$.

   - Compute $f_x = 9x^2y^2 - 10xy$.
   - Compute $f_y = 6x^3y - 5x^2$.
   - Differentiate f_x wrt x to get $f_{xx} = 18xy^2 - 10y$.
   - Differentiate f_y wrt y to get $f_{yy} = 6x^3$.
   - Differentiate f_x wrt y to get $f_{xy} = 18x^2y - 10x$.
   - Differentiate f_y wrt x to get $f_{yx} = 18x^2y - 10x$.

   Answer: f_xx = 18xy^2 - 10y, f_yy = 6x^3, f_xy = f_yx = 18x^2 y - 10x

2. Given $f(x, y) = e^{xy}$, show that $f_{xy} = f_{yx}$.

   - Compute $f_x = ye^{xy}$ using the chain rule.
   - Compute $f_y = xe^{xy}$ using the chain rule.
   - Compute f_{xy} by differentiating ye^{xy} wrt y using the product rule: $1 \cdot e^{xy} + y \cdot (xe^{xy}) = (1 + xy)e^{xy}$.
   - Compute f_{yx} by differentiating xe^{xy} wrt x using the product rule: $1 \cdot e^{xy} + x \cdot (ye^{xy}) = (1 + xy)e^{xy}$.

   Answer: f_xy = f_yx = (1 + xy)e^xy

3. Find the mixed partial derivative f_{xy} for $f(x, y) = \sec(x + y)$.

   - Compute $f_x = \sec(x + y)\tan(x + y)$ using the chain rule.
   - Differentiate f_x wrt y using the product rule: $\frac{\partial}{\partial y}[\sec(x+y)]\tan(x+y)+\sec(x+y)\frac{\partial}{\partial y}[\tan(x+y)]$.
   - The derivative of $\sec(u)$ is $\sec(u)\tan(u)u'$, and the derivative of $\tan(u)$ is $\sec^2(u)u'$.
   - Substitute $u = x + y$ and $u' = 1$: $[\sec(x + y)\tan(x + y)]\tan(x + y) + \sec(x + y)[\sec^2(x + y)]$.
   - Simplify to $\sec(x + y)\tan^2(x + y) + \sec^3(x + y)$.

   Answer: f_xy = \sec(x+y)\tan^2(x+y) + \sec^3(x+y)
