Cylindrical and spherical coordinates — three languages for space

Lesson 8 · Module 0 — Space, Surfaces & Curves · Multivariable & Vector Calculus · dailymaths.uk/vectorcalc

Practice problems

1. Convert the point $(0, 2, 2)$ from Cartesian coordinates to cylindrical coordinates (r, θ, z) and spherical coordinates (ρ, θ, ϕ).
2. Convert the equation of the sphere $x^2 + y^2 + z^2 = 16$ into spherical coordinates.
3. A surface is defined by the equation $\phi = \pi/6$ in spherical coordinates. Describe this surface in English and find its equation in cylindrical coordinates.

Solutions

1. Convert the point $(0, 2, 2)$ from Cartesian coordinates to cylindrical coordinates (r, θ, z) and spherical coordinates (ρ, θ, ϕ).

   - For cylindrical: $r = \sqrt{0^2 + 2^2} = 2$. Since $x = 0$ and $y = 2$, the point is on the positive y-axis, so $\theta = \pi/2$. z remains 2. Result: $(2, \pi/2, 2)$.
   - For spherical: $\rho = \sqrt{0^2 + 2^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$. θ is the same as cylindrical, $\pi/2$. For ϕ, $\cos(\phi) = z/\rho = 2/(2\sqrt{2}) = 1/\sqrt{2}$, so $\phi = \pi/4$. Result: $(2\sqrt{2}, \pi/2, \pi/4)$.

   Answer: Cylindrical: $(2, \pi/2, 2)$; Spherical: $(2\sqrt{2}, \pi/2, \pi/4)$

2. Convert the equation of the sphere $x^2 + y^2 + z^2 = 16$ into spherical coordinates.

   - Recall the definition of ρ in spherical coordinates: $\rho^2 = x^2 + y^2 + z^2$.
   - Substitute ρ^2 into the equation: $\rho^2 = 16$.
   - Taking the square root (since $\rho \geq 0$), we get $\rho = 4$.

   Answer: \rho $= 4$

3. A surface is defined by the equation $\phi = \pi/6$ in spherical coordinates. Describe this surface in English and find its equation in cylindrical coordinates.

   - The equation $\phi = \pi/6$ represents all points at a constant angle of 30 degrees from the positive z-axis, which is a right circular cone opening upwards.
   - In cylindrical coordinates, we know $\tan(\phi) = r/z$.
   - Substituting $\phi = \pi/6$, we get $\tan(\pi/6) = r/z$.
   - Since $\tan(\pi/6) = 1/\sqrt{3}$, the equation is $1/\sqrt{3} = r/z$, or $z = r\sqrt{3}$.

   Answer: A cone; $z = r\sqrt{3}$
