Surfaces II: Quadric Surfaces Gallery and Parametrised Surfaces

Lesson 7 · Module 0 — Space, Surfaces & Curves · Multivariable & Vector Calculus · daily-maths.uk/vectorcalc

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Practice problems

1. Identify the quadric surface given by the equation $x^2 + 2y^2 - z^2 = 6$.
2. Find a parametrisation $\vec{r}(u, v)$ for the sphere of radius 3 centered at the origin.
3. Parametrise the surface $z = x^2 - y^2$ for $0 \leq x \leq 1$ and $0 \leq y \leq 1$.

Solutions

1. Identify the quadric surface given by the equation $x^2 + 2y^2 - z^2 = 6$.
   - Divide the entire equation by 6 to get the standard form: $\frac{x^2}{6} + \frac{y^2}{3} - \frac{z^2}{6} = 1$.
   - Observe the signs of the squared terms: two are positive and one is negative.
   - Check the traces: setting $z = 0$ gives an ellipse $\frac{x^2}{6} + \frac{y^2}{3} = 1$; setting $x = 0$ gives a hyperbola $\frac{y^2}{3} - \frac{z^2}{6} = 1$.
   - A surface with one negative squared term and a constant of 1 is a hyperboloid of one sheet.

   Answer: Hyperboloid of one sheet

2. Find a parametrisation $\vec{r}(u, v)$ for the sphere of radius 3 centered at the origin.
   - Use spherical coordinates where u is the polar angle ϕ and v is the azimuthal angle θ.
   - The radius is fixed at $R = 3$.
   - The coordinates are $x = 3 \sin u \cos v$, $y = 3 \sin u \sin v$, and $z = 3 \cos u$.
   - Combine these into a vector function.

   Answer: \vecr(u, v) = \langle 3\sin u \cos v, 3\sin u \sin v, 3\cos u \rangle

3. Parametrise the surface $z = x^2 - y^2$ for $0 \le x \le 1$ and $0 \le y \le 1$.
   - Since the surface is given as a graph $z = f(x, y)$, use x and y as the parameters.
   - Let $u = x$ and $v = y$.
   - The vector components are $x(u, v) = u$, $y(u, v) = v$, and $z(u, v) = u^2 - v^2$.
   - Specify the bounds for the parameters based on the given constraints.

   Answer: \vecr(u, v) = \langle u, v, uˆ2 - vˆ2 \rangle, \text for 0 \le u \le 1, 0 \le v \le 1
