Surfaces I: graphs, level surfaces, and how to read them

Lesson 6 · Module 0 — Space, Surfaces & Curves · Multivariable & Vector Calculus · daily-maths.uk/vectorcalc

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Practice problems

1. Identify the surface described by the level surface $x^2 + 4y^2 + z^2 = 16$ and find the equation of its intersection with the xy-plane.
2. Given the function $f(x, y) = 4 - x^2 - y^2$, sketch the level curves for $k = 0, 1, 3, 4$ and describe the resulting surface.
3. Determine if the surface $z = \frac{1}{x^2+y^2}$ can be represented as a level surface of a function $F(x, y, z) = c$. If so, provide the function F.

Solutions

1. Identify the surface described by the level surface $x^2 + 4y^2 + z^2 = 16$ and find the equation of its intersection with the xy-plane.

   - Divide the entire equation by 16 to put it in standard form: $\frac{x^2}{16} + \frac{y^2}{4} + \frac{z^2}{16} = 1$.
   - Recognize this as the equation of an ellipsoid centered at the origin.
   - To find the intersection with the xy-plane, set $z = 0$.
   - The resulting equation is $x^2 + 4y^2 = 16$, which is an ellipse in the xy-plane.

   Answer: The surface is an ellipsoid; the intersection is the ellipse $x^2 + 4y^2 = 16$.

2. Given the function $f(x, y) = 4 - x^2 - y^2$, sketch the level curves for $k = 0, 1, 3, 4$ and describe the resulting surface.

   - For $k = 0$, $0 = 4 - x^2 - y^2 \implies x^2 + y^2 = 4$ (circle radius 2).
   - For $k = 1$, $1 = 4 - x^2 - y^2 \implies x^2 + y^2 = 3$ (circle radius $\sqrt{3}$).
   - For $k = 3$, $3 = 4 - x^2 - y^2 \implies x^2 + y^2 = 1$ (circle radius 1).
   - For $k = 4$, $4 = 4 - x^2 - y^2 \implies x^2 + y^2 = 0$ (the point $(0, 0)$).
   - The surface is a circular paraboloid opening downwards with its peak at $(0, 0, 4)$.

   Answer: The level curves are concentric circles with decreasing radii as k increases; the surface is a downward-opening circular paraboloid.

3. Determine if the surface $z = \frac{1}{x^2+y^2}$ can be represented as a level surface of a function $F(x, y, z) = c$. If so, provide the function F.

   - A graph $z = f(x, y)$ can always be rewritten as $z - f(x, y) = 0$.
   - Substitute the given function: $z - \frac{1}{x^2+y^2} = 0$.
   - Alternatively, multiply by $(x^2 + y^2)$ to clear the fraction: $z(x^2 + y^2) - 1 = 0$.
   - Define $F(x, y, z) = z(x^2 + y^2)$ and set the constant $c = 1$.

   Answer: Yes, it can be represented as $F(x, y, z) = z(x^2+y^2) = 1$ (or $F(x, y, z) = z - \frac{1}{x^2+y^2} = 0$).