Curves in space: vector-valued functions, velocity, acceleration

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Practice problems

1. Find the velocity vector $\vec{v}(t)$ and the acceleration vector $\vec{a}(t)$ for the position function $\vec{r}(t) = \langle t^3, \sin(t), \sqrt{t} \rangle$.
2. A particle moves along the curve $\vec{r}(t) = \langle e^{2t}, t^2, 5t \rangle$. Find the speed of the particle at time $t = 0$.
3. Determine the acceleration vector $\vec{a}(t)$ for a particle whose position is given by $\vec{r}(t) = \langle \cos(3t), \sin(3t), 2t \rangle$.

Solutions

1. Find the velocity vector $\vec{v}(t)$ and the acceleration vector $\vec{a}(t)$ for the position function $\vec{r}(t) = \langle t^3, \sin(t), \sqrt{t} \rangle$.

   - Differentiate the x-component: the derivative of t^3 is $3t^2$.
   - Differentiate the y-component: the derivative of $\sin(t)$ is $\cos(t)$.
   - Differentiate the z-component: the derivative of $\sqrt{t}$ is $\frac{1}{2\sqrt{t}}$.
   - Assemble the velocity vector: $\vec{v}(t) = \langle 3t^2, \cos(t), \frac{1}{2\sqrt{t}} \rangle$.
   - Differentiate the velocity components for acceleration: $3t^2$ becomes $6t$, $\cos(t)$ becomes $-\sin(t)$, and $\frac{1}{2}t^{-1/2}$ becomes $-\frac{1}{4}t^{-3/2}$.
   - Assemble the acceleration vector: $\vec{a}(t) = \langle 6t, -\sin(t), -\frac{1}{4t\sqrt{t}} \rangle$.

   Answer: \vecv(t) = \langle 3t^2, \cos(t), \frac12\sqrtt \rangle, \veca(t) = \langle 6t, -\sin(t), -\frac14t\sqrtt \rangle

2. A particle moves along the curve $\vec{r}(t) = \langle e^{2t}, t^2, 5t \rangle$. Find the speed of the particle at time $t = 0$.

   - Find the velocity vector $\vec{v}(t)$ by differentiating each component: $\vec{v}(t) = \langle 2e^{2t}, 2t, 5 \rangle$.
   - Evaluate the velocity at $t = 0$: $\vec{v}(0) = \langle 2e^0, 2(0), 5 \rangle = \langle 2, 0, 5 \rangle$.
   - Calculate the magnitude of the velocity vector: $|\vec{v}(0)| = \sqrt{2^2 + 0^2 + 5^2}$.
   - Simplify the result: $\sqrt{4 + 0 + 25} = \sqrt{29}$.

   Answer: \sqrt29

3. Determine the acceleration vector $\vec{a}(t)$ for a particle whose position is given by $\vec{r}(t) = \langle \cos(3t), \sin(3t), 2t \rangle$.

   - Find the velocity vector using the chain rule: $\vec{v}(t) = \langle -3\sin(3t), 3\cos(3t), 2 \rangle$.
   - Differentiate the velocity vector to find acceleration.
   - The derivative of $-3\sin(3t)$ is $-3 \cdot 3\cos(3t) = -9\cos(3t)$.
   - The derivative of $3\cos(3t)$ is $3 \cdot (-3\sin(3t)) = -9\sin(3t)$.
   - The derivative of the constant 2 is 0.
   - Assemble the acceleration vector: $\vec{a}(t) = \langle -9\cos(3t), -9\sin(3t), 0 \rangle$.

   Answer: \veca(t) = \langle -9\cos(3t), -9\sin(3t), 0 \rangle