3D coordinates, vectors, dot and cross products — rapid review

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Practice problems

1. Compute the dot product of $\vec{a} = \langle 3, -2, 5 \rangle$ and $\vec{b} = \langle 1, 2, 1 \rangle$. Are these vectors perpendicular?
2. Find the cross product $\vec{a} \times \vec{b}$ where $\vec{a} = \langle 2, 0, -1 \rangle$ and $\vec{b} = \langle 0, 3, 2 \rangle$.
3. Find a unit vector $\hat{n}$ that is perpendicular to both $\vec{u} = \langle 1, 0, 1 \rangle$ and $\vec{v} = \langle 0, 1, 1 \rangle$.

Solutions

1. Compute the dot product of $\vec{a} = \langle 3, -2, 5 \rangle$ and $\vec{b} = \langle 1, 2, 1 \rangle$. Are these vectors perpendicular?
   - Calculate the dot product: $(3)(1) + (-2)(2) + (5)(1)$
   - Simplify: $3 - 4 + 5 = 4$
   - Since the dot product is 4, which is not 0, the vectors are not perpendicular.

   Answer: 4, No

2. Find the cross product $\vec{a} \times \vec{b}$ where $\vec{a} = \langle 2, 0, -1 \rangle$ and $\vec{b} = \langle 0, 3, 2 \rangle$.
   - Set up the determinant: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 0 & -1 \\ 0 & 3 & 2 \end{vmatrix}$
   - i component: $(0)(2) - (-1)(3) = 3$
   - j component: $-((2)(2) - (-1)(0)) = -4$
   - k component: $(2)(3) - (0)(0) = 6$

   Answer: \langle 3, -4, 6 \rangle

3. Find a unit vector $\hat{n}$ that is perpendicular to both $\vec{u} = \langle 1, 0, 1 \rangle$ and $\vec{v} = \langle 0, 1, 1 \rangle$.
   - Find the cross product $\vec{n} = \vec{u} \times \vec{v} = \langle (0)(1) - (1)(1), -((1)(1) - (1)(0)), (1)(1) - (0)(0) \rangle = \langle -1, -1, 1 \rangle$
   - Calculate the magnitude: $|\vec{n}| = \sqrt{(-1)^2 + (-1)^2 + 1^2} = \sqrt{3}$
   - Normalize the vector: $\hat{n} = \frac{1}{\sqrt{3}} \langle -1, -1, 1 \rangle$

   Answer: \langle -\frac1\sqrt3, -\frac1\sqrt3, \frac1\sqrt3 \rangle

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