The one big idea: boundaries know what happens inside

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Practice problems

1. Describe the boundary of the solid cylinder defined by $x^2 + y^2 \leq 1$ and $0 \leq z \leq 1$.
2. If a flow represents the movement of water, and the total net flow through the boundary of a region is zero, what does the boundary interior principle suggest about the sources and sinks inside?
3. Consider a flat disk D in the xy-plane. If we know the flow along the boundary circle C is 5, and we create a hemispherical bowl S that has the same circle C as its rim, what is the total twist of the field through the bowl S?

Solutions

1. Describe the boundary of the solid cylinder defined by $x^2 + y^2 \leq 1$ and $0 \leq z \leq 1$.

   - The boundary consists of all points where the inequalities become equalities.
   - The side is the surface $x^2 + y^2 = 1$ for $0 \leq z \leq 1$.
   - The top is the disk $x^2 + y^2 \leq 1$ at $z = 1$.
   - The bottom is the disk $x^2 + y^2 \leq 1$ at $z = 0$.

   Answer: The boundary is the union of the lateral cylinder wall and the two circular end-caps.

2. If a flow represents the movement of water, and the total net flow through the boundary of a region is zero, what does the boundary interior principle suggest about the sources and sinks inside?

   - The boundary interior principle states that the total expansion inside equals the net flow through the boundary.
   - If the boundary flow is zero, the total integral of the expansion rate inside must be zero.
   - Expansion represents sources (positive) and sinks (negative).

   Answer: The total strength of sources inside the region exactly cancels the total strength of sinks.

3. Consider a flat disk D in the xy-plane. If we know the flow along the boundary circle C is 5, and we create a hemispherical bowl S that has the same circle C as its rim, what is the total twist of the field through the bowl S?

   - The boundary interior principle for surfaces states that the total twist through a surface equals the flow along its boundary.
   - The boundary of the bowl S is the circle C.
   - The flow along C is given as 5.

   Answer: The total twist through the bowl is 5.
