

Orthogonal Function Families

Lesson 85 · Module 6 — Series Solutions & Special Functions · Differential Equations · daily-maths.uk/diffeq

Practice problems

1. Determine if the functions $f(x) = x$ and $g(x) = x - 1$ are orthogonal on the interval $[0, 1]$ with weight function $w(x) = 1$.
2. Check if $f(x) = 1$ and $g(x) = x$ are orthogonal on the interval $[-1, 1]$ with weight function $w(x) = 1$.
3. Given the functions $f(x) = 1$ and $g(x) = 2x - 1$ on the interval $[0, 1]$, find the weight function $w(x) = c$ (a constant) that makes them orthogonal. If no such constant exists, explain why.

Solutions

1. Determine if the functions $f(x) = x$ and $g(x) = x - 1$ are orthogonal on the interval $[0, 1]$ with weight function $w(x) = 1$.

- Set up the inner product integral: $\int_0^1 (1)(x)(x - 1)dx$.
- Expand the integrand: $\int_0^1 (x^2 - x)dx$.
- Integrate term by term: $[\frac{1}{3}x^3 - \frac{1}{2}x^2]_0^1$.
- Evaluate at the boundaries: $(\frac{1}{3} - \frac{1}{2}) - (0 - 0) = -\frac{1}{6}$.
- Since the result is not zero, the functions are not orthogonal.

Answer: Not orthogonal

2. Check if $f(x) = 1$ and $g(x) = x$ are orthogonal on the interval $[-1, 1]$ with weight function $w(x) = 1$.

- Set up the inner product integral: $\int_{-1}^1 (1)(1)(x)dx$.
- Integrate x : $[\frac{1}{2}x^2]_{-1}^1$.
- Evaluate: $\frac{1}{2}(1)^2 - \frac{1}{2}(-1)^2 = \frac{1}{2} - \frac{1}{2} = 0$.
- Since the result is zero, the functions are orthogonal.

Answer: Orthogonal

3. Given the functions $f(x) = 1$ and $g(x) = 2x - 1$ on the interval $[0, 1]$, find the weight function $w(x) = c$ (a constant) that makes them orthogonal. If no such constant exists, explain why.

- Set up the weighted inner product: $\int_0^1 c(1)(2x - 1)dx = 0$.
- Factor out the constant: $c \int_0^1 (2x - 1)dx = 0$.
- Integrate: $c[x^2 - x]_0^1 = 0$.
- Evaluate: $c((1^2 - 1) - (0^2 - 0)) = c(0) = 0$.
- The integral is zero regardless of the value of c . Therefore, any constant $c > 0$ works.

Answer: Any constant $c > 0$