

Power series solutions II — Airy's equation

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Practice problems

1. Find the first four non-zero terms of the power series solution to $y'' = ty$ with initial conditions $y(0) = 0$ and $y'(0) = 1$.
2. For the solution to $y'' = ty$ with $y(0) = 1$ and $y'(0) = 0$, determine the value of the coefficient a_6 .
3. Show that if $y(t)$ is a solution to $y'' = ty$, then $u(t) = y(-t)$ is a solution to $u'' = -tu$.

Solutions

1. Find the first four non-zero terms of the power series solution to $y'' = ty$ with initial conditions $y(0) = 0$ and $y'(0) = 1$.

- Given $y(0) = 0$ and $y'(0) = 1$, we have $a_0 = 0$ and $a_1 = 1$.
- From the $k = 0$ term of the ODE, $2a_2 = 0$, so $a_2 = 0$.
- Using the recurrence $a_{k+2} = a_{k-1}/((k+2)(k+1))$, the non-zero terms depend on a_1 .
- For $k = 2$, $a_4 = a_1/(4 \cdot 3) = 1/12$.
- For $k = 5$, $a_7 = a_4/(7 \cdot 6) = (1/12)/42 = 1/504$.
- For $k = 8$, $a_{10} = a_7/(10 \cdot 9) = (1/504)/90 = 1/45360$.

Answer: $y(t) = t + \frac{1}{12}t^4 + \frac{1}{504}t^7 + \frac{1}{45360}t^{10} + \dots$

2. For the solution to $y'' = ty$ with $y(0) = 1$ and $y'(0) = 0$, determine the value of the coefficient a_6 .

- Given $a_0 = 1$ and $a_1 = 0$.
- The recurrence is $a_{k+2} = a_{k-1}/((k+2)(k+1))$.
- First non-zero term after a_0 is $a_3 = a_0/(3 \cdot 2) = 1/6$.
- The next non-zero term is $a_6 = a_3/(6 \cdot 5) = (1/6)/30 = 1/180$.

Answer: $a_6 = \frac{1}{180}$

3. Show that if $y(t)$ is a solution to $y'' = ty$, then $u(t) = y(-t)$ is a solution to $u'' = -tu$.

- Let $u(t) = y(-t)$.
- By the chain rule, $u'(t) = -y'(-t)$ and $u''(t) = (-1)^2 y''(-t) = y''(-t)$.
- Since y satisfies $y''(t) = ty(t)$, then $y''(-t) = (-t)y(-t)$.
- Substituting these into the expression for u , we get $u''(t) = -tu(t)$.

Answer: The substitution $u(t) = y(-t)$ transforms the equation $y'' = ty$ into $u'' = -tu$.