

The nonlinear pendulum — beyond small angles

Lesson 69 · Module 5 — Nonlinear Dynamics & Chaos · Differential Equations · dailymaths.uk/diffeq

Practice problems

1. For a pendulum where $\frac{g}{L} = 2$, given the initial conditions $y(0) = 0$ and $v(0) = 2$, determine if the motion is libration or rotation.
2. Find the energy constant C for a pendulum with $\frac{g}{L} = 1$ that starts at the unstable equilibrium $y = \pi$ with a small nudge of $v = 0.1$. Is this motion libration or rotation?
3. A pendulum with $\frac{g}{L} = 3$ is released from rest at an angle $y = \frac{\pi}{2}$. Determine if it will reach the top position $y = \pi$.

Solutions

1. For a pendulum where $\frac{g}{L} = 2$, given the initial conditions $y(0) = 0$ and $v(0) = 2$, determine if the motion is libration or rotation.

- Calculate the energy constant $C = \frac{1}{2}v^2 - \frac{g}{L}\cos(y)$.
- Substitute the values: $C = \frac{1}{2}(2)^2 - 2\cos(0) = 2 - 2(1) = 0$.
- The critical energy for the separatrix is $C_{crit} = \frac{g}{L} = 2$.
- Since $0 < 2$, the energy is below the critical threshold.

Answer: Libration

2. Find the energy constant C for a pendulum with $\frac{g}{L} = 1$ that starts at the unstable equilibrium $y = \pi$ with a small nudge of $v = 0.1$. Is this motion libration or rotation?

- Use the formula $C = \frac{1}{2}v^2 - \frac{g}{L}\cos(y)$.
- Substitute the values: $C = \frac{1}{2}(0.1)^2 - 1\cos(\pi) = 0.005 - (-1) = 1.005$.
- The critical energy is $C_{crit} = \frac{g}{L} = 1$.
- Since $1.005 > 1$, the energy is slightly above the separatrix.

Answer: Rotation

3. A pendulum with $\frac{g}{L} = 3$ is released from rest at an angle $y = \frac{\pi}{2}$. Determine if it will reach the top position $y = \pi$.

- Calculate the energy constant: $C = \frac{1}{2}(0)^2 - 3\cos(\frac{\pi}{2}) = 0 - 3(0) = 0$.
- The critical energy required to reach the top is $C_{crit} = \frac{g}{L} = 3$.
- Since $0 < 3$, the pendulum does not have enough energy to reach the top.
- The maximum angle y_{max} occurs when $v = 0$, so $0 - 3\cos(y_{max}) = 0$, meaning $\cos(y_{max}) = 0$, so $y_{max} = \frac{\pi}{2}$.

Answer: No, it will not reach the top.