

Competing species; coexistence vs exclusion

Lesson 68 · Module 5 — Nonlinear Dynamics & Chaos · Differential Equations · dailymaths.uk/diffeq

Practice problems

1. Given the system $x' = x(1 - \frac{x+0.4y}{50})$ and $y' = y(1 - \frac{y+0.3x}{100})$, find the x and y intercepts of both nullclines and determine if the species coexist stably.
2. For a system with $K_1 = 10, K_2 = 10, a_{12} = 2, a_{21} = 2$, sketch the nullclines and describe the stability of the interior equilibrium.
3. If species x is a 'super-competitor' such that $a_{12} = 0.1$ and $a_{21} = 5$ with $K_1 = K_2 = 10$, which species will survive regardless of initial populations?

Solutions

1. Given the system $x' = x(1 - \frac{x+0.4y}{50})$ and $y' = y(1 - \frac{y+0.3x}{100})$, find the x and y intercepts of both nullclines and determine if the species coexist stably.

- The x-nullcline is $x + 0.4y = 50$. Intercepts are $(50, 0)$ and $(0, 125)$.
- The y-nullcline is $y + 0.3x = 100$. Intercepts are $(333.33, 0)$ and $(0, 100)$.
- The carrying capacities are $K_1 = 50$ and $K_2 = 100$.
- Comparing intercepts: $50 < 333.33$ and $100 < 125$.
- Since the carrying capacities are the inner intercepts of the nullcline system, the interior equilibrium is stable.

Answer: Stable coexistence; equilibrium at $(30, 50)$.

2. For a system with $K_1 = 10, K_2 = 10, a_{12} = 2, a_{21} = 2$, sketch the nullclines and describe the stability of the interior equilibrium.

- x-nullcline: $x + 2y = 10$. Intercepts: $(10, 0), (0, 5)$.
- y-nullcline: $y + 2x = 10$. Intercepts: $(5, 0), (0, 10)$.
- The carrying capacities $(10, 0)$ and $(0, 10)$ are the outer intercepts.
- Checking the condition: $K_1 > K_2/a_{21} \implies 10 > 10/2 = 5$ and $K_2 > K_1/a_{12} \implies 10 > 10/2 = 5$.
- Both conditions for unstable coexistence are met, making the interior point a saddle.

Answer: Unstable coexistence (saddle point); competitive exclusion occurs.

3. If species x is a 'super-competitor' such that $a_{12} = 0.1$ and $a_{21} = 5$ with $K_1 = K_2 = 10$, which species will survive regardless of initial populations?

- x-nullcline: $x + 0.1y = 10$. Intercepts: $(10, 0), (0, 100)$.
- y-nullcline: $y + 5x = 10$. Intercepts: $(2, 0), (0, 10)$.
- The x-nullcline is entirely above and to the right of the y-nullcline in the first quadrant.
- Therefore, $x' > 0$ whenever $y' > 0$ is possible, and x will always drive y to zero.

Answer: Species x will always survive; species y is excluded.