

Complex eigenvalues — spirals and centres

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Practice problems

1. Classify the equilibrium point at the origin for the system $x' = -x + y, y' = -x - y$. Is it a stable spiral, unstable spiral, or a centre?
2. For the system $x' = 3x - 4y, y' = 4x + 3y$, what is the classification of the origin?
3. A system has a matrix A with $\text{Trace}(A) = 0$ and $\det(A) = 9$. Classify the equilibrium point and find the period of the orbits.

Solutions

1. Classify the equilibrium point at the origin for the system $x' = -x + y, y' = -x - y$. Is it a stable spiral, unstable spiral, or a centre?

- Write the matrix $A = \begin{pmatrix} -1 & 1 \\ -1 & -1 \end{pmatrix}$.
- Find the characteristic equation: $\det(A - \lambda I) = (-1 - \lambda)^2 + 1 = 0$.
- Solve for λ : $(-1 - \lambda)^2 = -1 \implies -1 - \lambda = \pm i \implies \lambda = -1 \pm i$.
- Identify the real part $\alpha = -1$.
- Since $\alpha < 0$, the origin is a stable spiral.

Answer: Stable Spiral

2. For the system $x' = 3x - 4y, y' = 4x + 3y$, what is the classification of the origin?

- Write the matrix $A = \begin{pmatrix} 3 & -4 \\ 4 & 3 \end{pmatrix}$.
- Find the characteristic equation: $(3 - \lambda)^2 + 16 = 0$.
- Solve for λ : $(3 - \lambda)^2 = -16 \implies 3 - \lambda = \pm 4i \implies \lambda = 3 \pm 4i$.
- Identify the real part $\alpha = 3$.
- Since $\alpha > 0$, the origin is an unstable spiral.

Answer: Unstable Spiral

3. A system has a matrix A with $\text{Trace}(A) = 0$ and $\det(A) = 9$. Classify the equilibrium point and find the period of the orbits.

- Since $\text{Trace}(A) = 0$, the real part $\alpha = 0$.
- The characteristic equation is $\lambda^2 - \text{Trace}(A)\lambda + \det(A) = 0$, so $\lambda^2 + 9 = 0$.
- The eigenvalues are $\lambda = \pm 3i$.
- Since the eigenvalues are purely imaginary, the origin is a centre.
- The imaginary part is $\beta = 3$. The period is $T = 2\pi/\beta = 2\pi/3$.

Answer: Centre with period $2\pi/3$