

From one equation to many

Lesson 51 · Module 4 — Systems of ODEs · Differential Equations · dailymaths.uk/diffeq

Practice problems

1. Find all equilibrium points for the competition system: $\frac{dx}{dt} = x(2 - x - y)$ and $\frac{dy}{dt} = y(3 - 2x - y)$.
2. A system is given by $x' = -x + 2y$ and $y' = 3x - 2y$. Write this in matrix form $\mathbf{x}' = A\mathbf{x}$.
3. For the system $x' = y$ and $y' = -x$, determine if the point $(0, 0)$ is an equilibrium and describe the trajectories qualitatively.

Solutions

1. Find all equilibrium points for the competition system: $\frac{dx}{dt} = x(2 - x - y)$ and $\frac{dy}{dt} = y(3 - 2x - y)$.
- Set $x(2 - x - y) = 0$. This implies $x = 0$ or $x + y = 2$.
 - Set $y(3 - 2x - y) = 0$. This implies $y = 0$ or $2x + y = 3$.
 - Case 1: $x = 0$ and $y = 0$. Point $(0, 0)$.
 - Case 2: $x = 0$ and $2x + y = 3 \implies y = 3$. Point $(0, 3)$.
 - Case 3: $x + y = 2$ and $y = 0 \implies x = 2$. Point $(2, 0)$.
 - Case 4: $x + y = 2$ and $2x + y = 3$. Subtracting the first from the second gives $x = 1$, then $y = 1$. Point $(1, 1)$.

Answer: $(0, 0)$, $(0, 3)$, $(2, 0)$, $(1, 1)$

2. A system is given by $x' = -x + 2y$ and $y' = 3x - 2y$. Write this in matrix form $\mathbf{x}' = A\mathbf{x}$.
- Identify the coefficients of x and y for the first equation: -1 and 2 .
 - Identify the coefficients of x and y for the second equation: 3 and -2 .
 - Place these into a 2×2 matrix A .

Answer: $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ 3 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

3. For the system $x' = y$ and $y' = -x$, determine if the point $(0, 0)$ is an equilibrium and describe the trajectories qualitatively.
- Check equilibrium: $f(0, 0) = 0$ and $g(0, 0) = -0 = 0$. Yes, it is an equilibrium.
 - Look at the velocity vector: $\vec{v} = (y, -x)$.
 - At $(1, 0)$, $\vec{v} = (0, -1)$ (down). At $(0, 1)$, $\vec{v} = (1, 0)$ (right). At $(-1, 0)$, $\vec{v} = (0, 1)$ (up). At $(0, -1)$, $\vec{v} = (-1, 0)$ (left).
 - The vectors point in a clockwise circle around the origin.

Answer: The point $(0, 0)$ is an equilibrium; trajectories are clockwise circles.