

Module 3 review + self-test

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Practice problems

1. Solve the initial value problem $y' + 3y = 4$ with $y(0) = 0$.
2. Find the solution to $y'' + 4y = \delta(t - 2)$ with $y(0) = 0, y'(0) = 0$.
3. Solve $y'' - 4y' + 4y = 0$ with $y(0) = 1, y'(0) = 2$.

Solutions

1. Solve the initial value problem $y' + 3y = 4$ with $y(0) = 0$.

- Transform the equation: $(s + 3)Y(s) = 4/s$.
- Solve for $Y(s)$: $Y(s) = 4/(s(s + 3))$.
- Partial fraction decomposition: $Y(s) = (4/3)/s - (4/3)/(s + 3)$.
- Inverse transform: $y(t) = 4/3 - (4/3)e^{-3t}$.

Answer: $y(t) = \frac{4}{3}(1 - e^{-3t})$

2. Find the solution to $y'' + 4y = \delta(t - 2)$ with $y(0) = 0, y'(0) = 0$.

- Transform the equation: $(s^2 + 4)Y(s) = e^{-2s}$.
- Solve for $Y(s)$: $Y(s) = e^{-2s}/(s^2 + 4)$.
- Identify the base transform: $\mathcal{L}^{-1}\{1/(s^2 + 4)\} = 0.5 \sin(2t)$.
- Apply the t-shift: $y(t) = 0.5 \sin(2(t - 2))u(t - 2)$.

Answer: $y(t) = 0.5 \sin(2(t-2)) u(t-2)$

3. Solve $y'' - 4y' + 4y = 0$ with $y(0) = 1, y'(0) = 2$.

- Transform the equation: $(s^2 - 4s + 4)Y(s) - sy(0) - y'(0) + 4y(0) = 0$.
- Substitute ICs: $(s - 2)^2 Y(s) - s - 2 + 4 = 0$.
- Solve for $Y(s)$: $Y(s) = (s - 2)/(s - 2)^2 = 1/(s - 2)$.
- Inverse transform: $y(t) = e^{2t}$.

Answer: $y(t) = e^{2t}$