Impulse response and why engineers love it

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Practice problems

1. Find the impulse response $h(t)$ for the system $y'' + 4y' + 3y = \delta(t)$ with $y(0) = 0$ and $y'(0) = 0$.
2. A system has a transfer function $H(s) = \frac{s}{s^2+16}$. Find the impulse response $h(t)$.
3. Find the impulse response for the system $y' + 2y = \delta(t)$ and use it to find the response to a constant unit step input $f(t) = u(t)$.

Solutions

1. Find the impulse response $h(t)$ for the system $y'' + 4y' + 3y = \delta(t)$ with $y(0) = 0$ and $y'(0) = 0$.

   - Take the Laplace transform: $s^2Y + 4sY + 3Y = 1$.
   - Solve for $Y(s)$: $H(s) = \frac{1}{s^2+4s+3}$.
   - Factor the denominator: $H(s) = \frac{1}{(s+1)(s+3)}$.
   - Use partial fractions: $\frac{1}{(s+1)(s+3)} = \frac{1}{2}\frac{1}{s+1} - \frac{1}{2}\frac{1}{s+3}$.
   - Invert the transform: $h(t) = \frac{1}{2}(e^{-t} - e^{-3t})u(t)$.

   Answer: h(t) = \frac12(e^-t - e^-3t)u(t)

2. A system has a transfer function $H(s) = \frac{s}{s^2+16}$. Find the impulse response $h(t)$.

   - The impulse response is the inverse Laplace transform of the transfer function: $h(t) = \mathcal{L}^{-1}\{\frac{s}{s^2+16}\}$.
   - Recognize the form $\frac{s}{s^2+\omega^2}$ as the transform of $\cos(\omega t)$.
   - Here $\omega = 4$.
   - Therefore, $h(t) = \cos(4t)u(t)$.

   Answer: h(t) = \cos(4t)u(t)

3. Find the impulse response for the system $y' + 2y = \delta(t)$ and use it to find the response to a constant unit step input $f(t) = u(t)$.

   - Find $h(t)$: $sY + 2Y = 1 \implies H(s) = \frac{1}{s+2} \implies h(t) = e^{-2t}u(t)$.
   - For input $f(t) = u(t)$, the transform is $F(s) = \frac{1}{s}$.
   - The output is $Y(s) = H(s)F(s) = \frac{1}{s(s+2)}$.
   - Partial fractions: $\frac{1}{s(s+2)} = \frac{1}{2}\frac{1}{s} - \frac{1}{2}\frac{1}{s+2}$.
   - Invert: $y(t) = \frac{1}{2}(1 - e^{-2t})u(t)$.

   Answer: h(t) = e^-2tu(t), y(t) = \frac12(1 - e^-2t)u(t)