

Shifting Theorems

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Practice problems

1. Find the Laplace transform of $f(t) = e^{-5t} \cos(2t)$.
2. Find the inverse Laplace transform of $G(s) = \frac{e^{-3s}}{s-2}$.
3. Find the Laplace transform of $f(t) = u(t-1)e^{t-1}$.

Solutions

1. Find the Laplace transform of $f(t) = e^{-5t} \cos(2t)$.

- Identify the base function $f_0(t) = \cos(2t)$.
- Find its transform $F_0(s) = \frac{s}{s^2+4}$.
- Apply the first shifting theorem with $a = -5$, replacing s with $s - (-5) = s + 5$.
- The result is $F(s) = \frac{s+5}{(s+5)^2+4}$.

Answer: $\frac{s+5}{(s+5)^2+4}$

2. Find the inverse Laplace transform of $G(s) = \frac{e^{-3s}}{s-2}$.

- Identify the part without the exponential: $H(s) = \frac{1}{s-2}$.
- Find the inverse transform of $H(s)$, which is $h(t) = e^{2t}$.
- Apply the second shifting theorem for $a = 3$.
- The result is $g(t) = u(t-3)h(t-3) = u(t-3)e^{2(t-3)}$.

Answer: $u(t-3)e^{2(t-3)}$

3. Find the Laplace transform of $f(t) = u(t-1)e^{t-1}$.

- Identify the base function $f_0(t) = e^t$.
- Find its transform $F_0(s) = \frac{1}{s-1}$.
- The function is given as $u(t-1)f_0(t-1)$, which fits the second shifting theorem exactly with $a = 1$.
- The transform is $e^{-1s}F_0(s) = \frac{e^{-s}}{s-1}$.

Answer: $\frac{e^{-s}}{s-1}$