Solving IVPs with Laplace — the full workflow

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Practice problems

1. Solve the initial value problem $y' + 3y = 0$ with $y(0) = 5$ using the Laplace transform workflow.
2. Solve the initial value problem $y' - 2y = 4$ with $y(0) = 1$ using the Laplace transform workflow.
3. Solve the initial value problem $y'' + 4y = 0$ with $y(0) = 0$ and $y'(0) = 2$ using the Laplace transform workflow.

Solutions

1. Solve the initial value problem $y' + 3y = 0$ with $y(0) = 5$ using the Laplace transform workflow.

   - Transform the equation: $sY(s) - y(0) + 3Y(s) = 0$.
   - Substitute $y(0) = 5$: $sY(s) - 5 + 3Y(s) = 0$.
   - Solve for $Y(s)$: $(s + 3)Y(s) = 5$, so $Y(s) = \frac{5}{s+3}$.
   - Invert using the table: $\mathcal{L}^{-1}\{\frac{5}{s+3}\} = 5e^{-3t}$.

   Answer: y(t) = 5 e^-3t

2. Solve the initial value problem $y' - 2y = 4$ with $y(0) = 1$ using the Laplace transform workflow.

   - Transform the equation: $sY(s) - y(0) - 2Y(s) = \frac{4}{s}$.
   - Substitute $y(0) = 1$: $(s - 2)Y(s) - 1 = \frac{4}{s}$.
   - Isolate $Y(s)$: $(s - 2)Y(s) = \frac{4}{s} + 1 = \frac{s+4}{s}$, so $Y(s) = \frac{s+4}{s(s-2)}$.
   - Partial fractions: $\frac{s+4}{s(s-2)} = \frac{A}{s} + \frac{B}{s-2}$. Solving gives $A = -2$ and $B = 3$.
   - Invert: $y(t) = -2(1) + 3e^{2t}$.

   Answer: y(t) = 3 e^2t - 2

3. Solve the initial value problem $y'' + 4y = 0$ with $y(0) = 0$ and $y'(0) = 2$ using the Laplace transform workflow.

   - Transform the equation: $s^2Y(s) - sy(0) - y'(0) + 4Y(s) = 0$.
   - Substitute $y(0) = 0$ and $y'(0) = 2$: $s^2Y(s) - 2 + 4Y(s) = 0$.
   - Solve for $Y(s)$: $(s^2 + 4)Y(s) = 2$, so $Y(s) = \frac{2}{s^2+4}$.
   - Recognize the form $\frac{a}{s^2+a^2}$ where $a = 2$.
   - Invert: $y(t) = \sin(2t)$.

   Answer: y(t) = \sin(2t)
