Transform rules and the essential table

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Practice problems

1. Compute the Laplace transform of $f(t) = 3e^{2t} + 4\sin(3t)$.
2. Given $y' - 2y = 0$ with $y(0) = 5$, find the algebraic expression for $Y(s)$.
3. Transform the ODE $y'' + 4y = 0$ with $y(0) = 0$ and $y'(0) = 2$ into an equation for $Y(s)$ and solve for $Y(s)$.

Solutions

1. Compute the Laplace transform of $f(t) = 3e^{2t} + 4\sin(3t)$.

   - Apply linearity: $\mathcal{L}\{3e^{2t} + 4\sin(3t)\} = 3\mathcal{L}\{e^{2t}\} + 4\mathcal{L}\{\sin(3t)\}$.
   - Use the table for e^{at} with $a = 2$: $\mathcal{L}\{e^{2t}\} = \frac{1}{s-2}$.
   - Use the table for $\sin(at)$ with $a = 3$: $\mathcal{L}\{\sin(3t)\} = \frac{3}{s^2+9}$.
   - Combine the results: $3\frac{1}{s-2} + 4\frac{3}{s^2+9}$.

   Answer: \frac3s-2 + \frac12sˆ2+9

2. Given $y' - 2y = 0$ with $y(0) = 5$, find the algebraic expression for $Y(s)$.

   - Transform both sides: $\mathcal{L}\{y'\} - 2\mathcal{L}\{y\} = \mathcal{L}\{0\}$.
   - Substitute the derivative rule: $(sY - y(0)) - 2Y = 0$.
   - Plug in the initial condition $y(0) = 5$: $sY - 5 - 2Y = 0$.
   - Factor out Y: $Y(s - 2) = 5$.
   - Solve for Y: $Y(s) = \frac{5}{s-2}$.

   Answer: Y(s) = \frac5s-2

3. Transform the ODE $y'' + 4y = 0$ with $y(0) = 0$ and $y'(0) = 2$ into an equation for $Y(s)$ and solve for $Y(s)$.

   - Transform the equation: $\mathcal{L}\{y''\} + 4\mathcal{L}\{y\} = 0$.
   - Apply the second derivative rule: $(s^2Y - sy(0) - y'(0)) + 4Y = 0$.
   - Substitute $y(0) = 0$ and $y'(0) = 2$: $s^2Y - s(0) - 2 + 4Y = 0$.
   - Simplify: $s^2Y + 4Y = 2$.
   - Factor Y: $Y(s^2 + 4) = 2$.
   - Solve for Y: $Y(s) = \frac{2}{s^2+4}$.

   Answer: Y(s) = \frac2sˆ2+4
