RLC circuits — the mechanical-electrical analogy

Practice problems

1. An RLC circuit has $L = 2$ H, $R = 10$ Ω, and $C = 0.1$ F. Write the homogeneous differential equation for the charge $q(t)$ and determine if the system is underdamped, overdamped, or critically damped.
2. Find the resonant frequency ω_0 for a circuit with $L = 0.5$ H and $C = 2 \times 10^{-6}$ F.
3. A circuit is critically damped with $L = 1$ H and $C = 0.25$ F. What is the value of the resistance R?

Solutions

1. An RLC circuit has $L = 2$ H, $R = 10\ \Omega$, and $C = 0.1$ F. Write the homogeneous differential equation for the charge $q(t)$ and determine if the system is underdamped, overdamped, or critically damped.

   - The equation is $Lq'' + Rq' + \frac{1}{C}q = 0$.
   - Substitute values: $2q'' + 10q' + \frac{1}{0.1}q = 0$, which simplifies to $2q'' + 10q' + 10q = 0$.
   - The characteristic equation is $2r^2 + 10r + 10 = 0$, or $r^2 + 5r + 5 = 0$.
   - The discriminant is $D = 5^2 - 4(1)(5) = 25 - 20 = 5$.
   - Since $D > 0$, the roots are real and distinct, meaning the system is overdamped.

   Answer: 2q" + 10q' + 10q = 0; overdamped

2. Find the resonant frequency ω_0 for a circuit with $L = 0.5$ H and $C = 2 \times 10^{-6}$ F.

   - Use the formula $\omega_0 = \frac{1}{\sqrt{LC}}$.
   - Calculate $LC = 0.5 \times 2 \times 10^{-6} = 1 \times 10^{-6}$.
   - Take the square root: $\sqrt{10^{-6}} = 10^{-3}$.
   - Compute the reciprocal: $\omega_0 = \frac{1}{10^{-3}} = 1000$.

   Answer: 1000 \text rad/s

3. A circuit is critically damped with $L = 1$ H and $C = 0.25$ F. What is the value of the resistance R?

   - For critical damping, the discriminant of $Lr^2 + Rr + \frac{1}{C} = 0$ must be zero.
   - This requires $R^2 - 4L\frac{1}{C} = 0$.
   - Substitute $L = 1$ and $C = 0.25$: $R^2 - 4(1)(\frac{1}{0.25}) = 0$.
   - Simplify: $R^2 - 4(4) = 0 \implies R^2 = 16$.
   - Since resistance must be positive, $R = 4$.

   Answer: 4 \Omega
