Forced oscillations and resonance II — beats and the resonance curve

Practice problems

1. A system has a natural frequency of $\omega_0 = 5$ rad/s. It is driven by a force with frequency $\omega = 4.8$ rad/s. Calculate the beat period T_{beat}.
2. For the ODE $y'' + 9y = \sin(2.9t)$, find the amplitude of the steady-state (particular) solution.
3. A system with $\omega_0 = 2$ is driven by $F_0 \sin(\omega t)$. If the desired steady-state amplitude is $A = 10$, what forcing frequency ω (where $\omega < \omega_0$) should be used, assuming $F_0 = 0.1$?

Solutions

1. A system has a natural frequency of $\omega_0 = 5$ rad/s. It is driven by a force with frequency $\omega = 4.8$ rad/s. Calculate the beat period T_{beat}.

   - Identify the frequency difference: $|\omega_0 - \omega| = |5 - 4.8| = 0.2$ rad/s.
   - Use the beat period formula: $T_{beat} = \frac{2\pi}{|\omega_0 - \omega|}$.
   - Compute the value: $T_{beat} = \frac{2\pi}{0.2} = 10\pi$.

   Answer: 10\pi \approx 31.42 \text s

2. For the ODE $y'' + 9y = \sin(2.9t)$, find the amplitude of the steady-state (particular) solution.

   - Identify $\omega_0^2 = 9$ and $\omega = 2.9$.
   - The particular solution is $y_p(t) = \frac{1}{9 - 2.9^2} \sin(2.9t)$.
   - Calculate the denominator: $9 - 8.41 = 0.59$.
   - The amplitude is $A = |1/0.59|$.

   Answer: \approx 1.695

3. A system with $\omega_0 = 2$ is driven by $F_0 \sin(\omega t)$. If the desired steady-state amplitude is $A = 10$, what forcing frequency ω (where $\omega < \omega_0$) should be used, assuming $F_0 = 0.1$?

   - Set up the amplitude equation: $10 = \frac{0.1}{4 - \omega^2}$.
   - Rearrange to solve for the denominator: $4 - \omega^2 = \frac{0.1}{10} = 0.01$.
   - Solve for ω^2: $\omega^2 = 4 - 0.01 = 3.99$.
   - Take the square root: $\omega = \sqrt{3.99}$.

   Answer: \approx 1.997 \text rad/s
