The harmonic oscillator I — mass on a spring

Practice problems

1. A mass of $m = 2$ kg is attached to a spring with $k = 8$ N/m. The mass is pulled 3 units from equilibrium and released from rest. Find the equation of motion $y(t)$ and the period T.

2. A mass of $m = 1$ kg and spring constant $k = 25$ N/m is pushed from equilibrium with an initial velocity of 10 units/s. Find the equation of motion $y(t)$ and the period T.

3. A mass of $m = 1$ kg and $k = 16$ N/m is pulled 2 units and pushed with a velocity of 4 units/s. Find the equation of motion $y(t)$.

Solutions

1. A mass of $m = 2$ kg is attached to a spring with $k = 8$ N/m. The mass is pulled 3 units from equilibrium and released from rest. Find the equation of motion $y(t)$ and the period T.

   - Calculate $\omega = \sqrt{k/m} = \sqrt{8/2} = 2$.
   - General solution: $y(t) = c_1 \cos(2t) + c_2 \sin(2t)$.
   - Apply $y(0) = 3$: $c_1(1) + c_2(0) = 3 \implies c_1 = 3$.
   - Apply $y'(0) = 0$: $y'(t) = -2c_1 \sin(2t) + 2c_2 \cos(2t) \implies 2c_2 = 0 \implies c_2 = 0$.
   - Calculate $T = 2\pi/\omega = 2\pi/2 = \pi$.

   Answer: $y(t) = 3\cos(2t), T = \pi$

2. A mass of $m = 1$ kg and spring constant $k = 25$ N/m is pushed from equilibrium with an initial velocity of 10 units/s. Find the equation of motion $y(t)$ and the period T.

   - Calculate $\omega = \sqrt{25/1} = 5$.
   - General solution: $y(t) = c_1 \cos(5t) + c_2 \sin(5t)$.
   - Apply $y(0) = 0$: $c_1 = 0$.
   - Apply $y'(0) = 10$: $5c_2 \cos(0) = 10 \implies c_2 = 2$.
   - Calculate $T = 2\pi/\omega = 2\pi/5$.

   Answer: $y(t) = 2\sin(5t), T = 0.4\pi$

3. A mass of $m = 1$ kg and $k = 16$ N/m is pulled 2 units and pushed with a velocity of 4 units/s. Find the equation of motion $y(t)$.

   - Calculate $\omega = \sqrt{16/1} = 4$.
   - General solution: $y(t) = c_1 \cos(4t) + c_2 \sin(4t)$.
   - Apply $y(0) = 2$: $c_1 = 2$.
   - Apply $y'(0) = 4$: $y'(t) = -4c_1 \sin(4t) + 4c_2 \cos(4t) \implies 4c_2 = 4 \implies c_2 = 1$.

   Answer: $y(t) = 2\cos(4t) + \sin(4t)$
