Distinct real roots — overdamped worlds

Lesson 26 · Module 2 — Second-Order Linear ODEs · Differential Equations · dailymaths.uk/diffeq

Practice problems

1. Solve the initial value problem $y'' + 5y' + 6y = 0$ with $y(0) = 2$ and $y'(0) = 1$.
2. Find the general solution for $y'' - 4y = 0$ and describe the behavior as t approaches infinity.
3. A system is described by $y'' + 2y' = 0$ with $y(0) = 1$ and $y'(0) = -1$. Solve for $y(t)$.

Solutions

1. Solve the initial value problem $y'' + 5y' + 6y = 0$ with $y(0) = 2$ and $y'(0) = 1$.

   - The characteristic equation is $r^2 + 5r + 6 = 0$, which factors as $(r + 2)(r + 3) = 0$.
   - The roots are $r_1 = -2$ and $r_2 = -3$, so the general solution is $y(t) = c_1e^{-2t} + c_2e^{-3t}$.
   - Using $y(0) = 2$, we get $c_1 + c_2 = 2$.
   - The derivative is $y'(t) = -2c_1e^{-2t} - 3c_2e^{-3t}$. Using $y'(0) = 1$, we get $-2c_1 - 3c_2 = 1$.
   - Solving the system: $2(c_1 + c_2) = 4$, so $(-2c_1 - 3c_2) + (2c_1 + 2c_2) = 1 + 4$, which gives $-c_2 = 5$, so $c_2 = -5$.
   - Then $c_1 = 2 - (-5) = 7$.

   Answer: $y(t) = 7e^{-2t} - 5e^{-3t}$

2. Find the general solution for $y'' - 4y = 0$ and describe the behavior as t approaches infinity.

   - The characteristic equation is $r^2 - 4 = 0$, so $r^2 = 4$.
   - The roots are $r = 2$ and $r = -2$.
   - The general solution is $y(t) = c_1e^{2t} + c_2e^{-2t}$.
   - As t approaches infinity, the e^{2t} term grows without bound (unless $c_1 = 0$), so the solution is unstable.

   Answer: $y(t) = c_1e^{2t} + c_2e^{-2t}$; unstable growth

3. A system is described by $y'' + 2y' = 0$ with $y(0) = 1$ and $y'(0) = -1$. Solve for $y(t)$.

   - The characteristic equation is $r^2 + 2r = 0$, which factors as $r(r + 2) = 0$.
   - The roots are $r_1 = 0$ and $r_2 = -2$.
   - The general solution is $y(t) = c_1e^{0t} + c_2e^{-2t} = c_1 + c_2e^{-2t}$.
   - Using $y(0) = 1$, we get $c_1 + c_2 = 1$.
   - The derivative is $y'(t) = -2c_2e^{-2t}$. Using $y'(0) = -1$, we get $-2c_2 = -1$, so $c_2 = 1/2$.
   - Then $c_1 = 1 - 1/2 = 1/2$.

   Answer: $y(t) = \frac{1}{2} + \frac{1}{2}e^{-2t}$
