Why second order rules physics: F = ma as an ODE

Practice problems

1. A mass of 5 kg is attached to a spring with a constant $k = 20$ N/m. The system experiences a damping force proportional to velocity with $c = 3$ kg/s and is driven by an external force $f(t) = 5\cos(2t)$. Write the second order ODE for the position $x(t)$.

2. A 2 kg mass is on a frictionless surface attached to a spring with $k = 18$ N/m. There is no external force. Write the ODE and verify if $x(t) = \sin(3t)$ is a solution.

3. A system is described by the ODE $x'' + 4x' + 4x = 0$. Describe the physical components of this system in terms of mass, damping, and spring constant, assuming $m = 1$ kg.

Solutions

1. A mass of 5 kg is attached to a spring with a constant $k = 20$ N/m. The system experiences a damping force proportional to velocity with $c = 3$ kg/s and is driven by an external force $f(t) = 5\cos(2t)$. Write the second order ODE for the position $x(t)$.

   - Identify constants: $m = 5$, $c = 3$, $k = 20$.
   - Apply Newton's second law: $mx'' = -cx' - kx + f(t)$.
   - Substitute values: $5x'' = -3x' - 20x + 5\cos(2t)$.
   - Rearrange to standard form: $5x'' + 3x' + 20x = 5\cos(2t)$.

   Answer: 5 x" + 3 x' + 20 x = 5\cos(2t)

2. A 2 kg mass is on a frictionless surface attached to a spring with $k = 18$ N/m. There is no external force. Write the ODE and verify if $x(t) = \sin(3t)$ is a solution.

   - Equation: $2x'' = -18x \implies x'' + 9x = 0$.
   - First derivative: $x'(t) = 3\cos(3t)$.
   - Second derivative: $x''(t) = -9\sin(3t)$.
   - Substitute into ODE: $-9\sin(3t) + 9\sin(3t) = 0$.
   - The equation holds.

   Answer: x" + 9 x = 0; Yes, it is a solution.

3. A system is described by the ODE $x'' + 4x' + 4x = 0$. Describe the physical components of this system in terms of mass, damping, and spring constant, assuming $m = 1$ kg.

   - Compare $x'' + 4x' + 4x = 0$ to $mx'' + cx' + kx = 0$.
   - Given $m = 1$, we see $c = 4$ and $k = 4$.
   - The system consists of a 1 kg mass, a damping coefficient of 4 kg/s, and a spring constant of 4 N/m.

   Answer: m = 1\text kg, c = 4\text kg/s, k = 4\text N/m
