Compound interest, loans, and mortgages as ODEs

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Practice problems

1. A loan of $15,000$ is taken at an annual interest rate of 6% with a constant repayment of $2,000$ per year. Find the balance after 5 years.
2. How long will it take to pay off a $50,000$ loan at 4% interest if the annual repayment is $6,000$?
3. You start a savings account with $5,000$ at 3% interest and deposit $1,200$ per year. What is the balance after 20 years?

Solutions

1. A loan of $15,000$ is taken at an annual interest rate of 6% with a constant repayment of $2,000$ per year. Find the balance after 5 years.

   - Identify constants: $B_0 = 15000$, $r = 0.06$, $P = 2000$.
   - Calculate equilibrium: $P/r = 2000/0.06 = 33333.33$.
   - Use formula: $B(t) = 33333.33 + (15000 - 33333.33)e^{0.06t}$.
   - Substitute $t = 5$: $B(5) = 33333.33 - 18333.33e^{0.3}$.
   - Compute: $B(5) \approx 33333.33 - 18333.33(1.3498) \approx 8586.33$.

   Answer: $8,586.33$

2. How long will it take to pay off a $50,000$ loan at 4% interest if the annual repayment is $6,000$?

   - Identify constants: $B_0 = 50000$, $r = 0.04$, $P = 6000$.
   - Use the payoff time formula: $T = \frac{1}{0.04} \ln\left(\frac{6000}{6000 - 0.04(50000)}\right)$.
   - Simplify denominator: $6000 - 2000 = 4000$.
   - Calculate ratio: $6000/4000 = 1.5$.
   - Compute $T = 25 \ln(1.5) \approx 25(0.4055) \approx 10.14$.

   Answer: 10.14 years

3. You start a savings account with $5,000$ at 3% interest and deposit $1,200$ per year. What is the balance after 20 years?

   - Identify constants: $B_0 = 5000$, $r = 0.03$, $P = -1200$ (since it is a deposit).
   - The ODE is $B' = 0.03B + 1200$.
   - Equilibrium $P/r = -1200/0.03 = -40000$.
   - Use formula: $B(t) = -40000 + (5000 - (-40000))e^{0.03t} = -40000 + 45000e^{0.03t}$.
   - Substitute $t = 20$: $B(20) = -40000 + 45000e^{0.6} \approx -40000 + 45000(1.8221) \approx 41994.50$.

   Answer: $41,994.50$
