RC Circuits

Practice problems

1. A circuit consists of a 12 V battery, a 200Ω resistor, and a 0.01 F capacitor. Find the time constant τ and the charge $q(t)$ at $t = 3$ s if the capacitor starts empty.

2. A capacitor with an initial charge of 0.8 C is discharged through a resistor of 100Ω and a capacitor of 0.05 F. How long does it take for the charge to drop to 0.2 C?

3. In a charging RC circuit, the charge reaches 0.05 C at $t = 1$ s and 0.08 C at $t = 2$ s, starting from $q(0) = 0$. Find the time constant τ and the source voltage V if $R = 100\Omega$.

Solutions

1. A circuit consists of a 12 V battery, a 200Ω resistor, and a 0.01 F capacitor. Find the time constant τ and the charge $q(t)$ at $t = 3$ s if the capacitor starts empty.

   - Calculate $\tau = RC = 200 \times 0.01 = 2$ s.
   - Calculate $Q_{max} = CV = 0.01 \times 12 = 0.12$ C.
   - Use the charging formula $q(t) = 0.12(1 - e^{-t/2})$.
   - Substitute $t = 3$: $q(3) = 0.12(1 - e^{-1.5}) \approx 0.12(1 - 0.223) \approx 0.0932$ C.

   Answer: $\tau = 2$ s, $q(3) \approx 0.0932$ C

2. A capacitor with an initial charge of 0.8 C is discharged through a resistor of 100Ω and a capacitor of 0.05 F. How long does it take for the charge to drop to 0.2 C?

   - Calculate $\tau = RC = 100 \times 0.05 = 5$ s.
   - Set up the discharge equation: $0.2 = 0.8e^{-t/5}$.
   - Divide by 0.8: $0.25 = e^{-t/5}$.
   - Take the natural log: $\ln(0.25) = -t/5$.
   - Solve for t: $t = -5\ln(0.25) = 5\ln(4) \approx 6.93$ s.

   Answer: $t \approx 6.93$ s

3. In a charging RC circuit, the charge reaches 0.05 C at $t = 1$ s and 0.08 C at $t = 2$ s, starting from $q(0) = 0$. Find the time constant τ and the source voltage V if $R = 100\Omega$.

   - Use the ratio $q(2)/q(1) = (1 - e^{-2/\tau})/(1 - e^{-1/\tau})$.
   - Let $u = e^{-1/\tau}$. Then $0.08/0.05 = (1 - u^2)/(1 - u) = 1 + u$.
   - Solve for u: $1.6 = 1 + u \implies u = 0.6$.
   - Solve for τ: $e^{-1/\tau} = 0.6 \implies -1/\tau = \ln(0.6) \implies \tau = -1/\ln(0.6) \approx 2.23$ s.
   - Find C: $C = \tau/R = 2.23/100 = 0.0223$ F.
   - Find Q_{max}: $0.05 = Q_{max}(1 - 0.6) \implies Q_{max} = 0.05/0.4 = 0.125$ C.
   - Find V: $V = Q_{max}/C = 0.125/0.0223 \approx 5.61$ V.

   Answer: $\tau \approx 2.23$ s, $V \approx 5.61$ V
