Falling with air resistance; terminal velocity

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Practice problems

1. A spherical bead with mass $m = 0.001$ kg falls through oil with a drag coefficient $c = 0.0002$ kg/s. Using $g = 9.8$ m/s^2, find the terminal velocity and the expression for $v(t)$ if the bead is dropped from rest.
2. An object reaches 90% of its terminal velocity in 10 seconds. Find the value of the constant $k = c/m$.
3. A projectile of mass m is thrown downwards with an initial velocity v_0 that is exactly twice its terminal velocity v_{term}. Show that the velocity at any time t is given by $v(t) = v_{term}(1 + e^{-kt})$.

Solutions

1. A spherical bead with mass $m = 0.001$ kg falls through oil with a drag coefficient $c = 0.0002$ kg/s. Using $g = 9.8$ m/s^2, find the terminal velocity and the expression for $v(t)$ if the bead is dropped from rest.

   - Calculate $k = c/m = 0.0002/0.001 = 0.2$ s^{-1}.
   - Calculate $v_{term} = g/k = 9.8/0.2 = 49$ m/s.
   - Use the formula $v(t) = v_{term}(1 - e^{-kt})$ for an object dropped from rest.
   - Substitute the values: $v(t) = 49(1 - e^{-0.2t})$.

   Answer: v_term = 49 \text m/s, v(t) = 49(1 - e^-0.2t)

2. An object reaches 90% of its terminal velocity in 10 seconds. Find the value of the constant $k = c/m$.

   - The velocity for an object dropped from rest is $v(t) = v_{term}(1 - e^{-kt})$.
   - Set $v(10) = 0.9v_{term}$, so $0.9v_{term} = v_{term}(1 - e^{-10k})$.
   - Divide by v_{term} to get $0.9 = 1 - e^{-10k}$, which simplifies to $e^{-10k} = 0.1$.
   - Take the natural logarithm of both sides: $-10k = \ln(0.1)$.
   - Solve for k: $k = -\ln(0.1)/10 \approx 2.303/10 = 0.2303$ s^{-1}.

   Answer: k \approx 0.2303 \text s^-1

3. A projectile of mass m is thrown downwards with an initial velocity v_0 that is exactly twice its terminal velocity v_{term}. Show that the velocity at any time t is given by $v(t) = v_{term}(1 + e^{-kt})$.

   - The general solution to $v' = g - kv$ is $v(t) = v_{term} + Ce^{-kt}$.
   - We are given $v(0) = 2v_{term}$.
   - Substitute $t = 0$ into the general solution: $2v_{term} = v_{term} + Ce^0$.
   - Solve for C: $C = 2v_{term} - v_{term} = v_{term}$.
   - Substitute C back into the general solution: $v(t) = v_{term} + v_{term}e^{-kt}$.
   - Factor out v_{term} to get $v(t) = v_{term}(1 + e^{-kt})$.

   Answer: v(t) = v_term(1 + e^-kt)
