

Harvesting and critical thresholds

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Practice problems

1. A population follows the model $y' = 0.5y(1 - y/800) - 50$. Find the two equilibrium populations and identify which is stable and which is the collapse threshold.
2. For a population with $r = 0.2$ and $K = 5000$, what is the maximum sustainable harvest rate H_{max} ? What happens if $H = 260$?
3. A manager wants to maintain a population at exactly $y = 400$ with $r = 1$ and $K = 1000$. What constant harvest rate H should be set, and is this equilibrium stable?

Solutions

1. A population follows the model $y' = 0.5y(1 - y/800) - 50$. Find the two equilibrium populations and identify which is stable and which is the collapse threshold.

- Set $0.5y(1 - y/800) - 50 = 0$.
- Multiply by 2 to simplify: $y(1 - y/800) - 100 = 0$.
- Expand: $y - y^2/800 - 100 = 0$.
- Multiply by -800 : $y^2 - 800y + 80000 = 0$.
- Use quadratic formula: $y = [800 \pm \sqrt{640000 - 320000}]/2 = [800 \pm \sqrt{320000}]/2$.
- Calculate: $y = 400 \pm 200\sqrt{2}$, so $y \approx 682.8$ and $y \approx 117.2$.

Answer: Stable equilibrium at $y \approx 682.8$, collapse threshold at $y \approx 117.2$.

2. For a population with $r = 0.2$ and $K = 5000$, what is the maximum sustainable harvest rate H_{max} ? What happens if $H = 260$?

- Use the formula $H_{max} = rK/4$.
- Substitute values: $H_{max} = (0.2 \cdot 5000)/4 = 1000/4 = 250$.
- Compare $H = 260$ to $H_{max} = 250$.
- Since $260 > 250$, the harvest rate exceeds the maximum growth rate of the population.

Answer: $H_{max} = 250$; if $H = 260$, the population will collapse regardless of the initial size.

3. A manager wants to maintain a population at exactly $y = 400$ with $r = 1$ and $K = 1000$. What constant harvest rate H should be set, and is this equilibrium stable?

- Set $y' = 0$ at $y = 400$: $1(400)(1 - 400/1000) - H = 0$.
- Calculate: $400(1 - 0.4) - H = 400(0.6) - H = 240 - H = 0$.
- Solve for H : $H = 240$.
- Check stability: The growth parabola $y(1 - y/1000)$ peaks at $y = 500$. Since $400 < 500$, this is the lower equilibrium point.

Answer: $H = 240$; the equilibrium is unstable (it is the collapse threshold).