The logistic equation — growth with limits

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Practice problems

1. A bacteria colony grows according to the logistic equation with a carrying capacity of $K = 5000$ and an intrinsic growth rate of $r = 0.2$ per hour. If the initial population is $y(0) = 500$, find the population after $t = 5$ hours.
2. A population of deer in a forest is modeled by $y' = 0.05y(1 - y/2000)$. If the current population is 2500, will the population increase or decrease? Find the population after 10 years.
3. A species of plant is introduced to an island. The population grows logistically. After 1 year, the population is 100. After 2 years, it is 400. If the carrying capacity is 1000, find the intrinsic growth rate r.

Solutions

1. A bacteria colony grows according to the logistic equation with a carrying capacity of $K = 5000$ and an intrinsic growth rate of $r = 0.2$ per hour. If the initial population is $y(0) = 500$, find the population after $t = 5$ hours.

   - Identify constants: $K = 5000$, $r = 0.2$, $y_0 = 500$.
   - Solve for C: $500 = 5000/(1 + C) \implies 1 + C = 10 \implies C = 9$.
   - Write the solution: $y(t) = 5000/(1 + 9e^{-0.2t})$.
   - Calculate $y(5) = 5000/(1 + 9e^{-0.2\cdot 5}) = 5000/(1 + 9e^{-1}) \approx 5000/(1 + 3.31) \approx 1160$.

   Answer: $y(5) \approx 1160$

2. A population of deer in a forest is modeled by $y' = 0.05y(1 - y/2000)$. If the current population is 2500, will the population increase or decrease? Find the population after 10 years.

   - Since $y(0) = 2500$ is greater than $K = 2000$, the term $(1-y/K)$ is negative, so y' is negative and the population decreases.
   - Solve for C: $2500 = 2000/(1 + C) \implies 1 + C = 0.8 \implies C = -0.2$.
   - Use the formula $y(t) = 2000/(1 - 0.2e^{-0.05t})$.
   - For $t = 10$: $y(10) = 2000/(1 - 0.2e^{-0.5}) \approx 2000/(1 - 0.2 \cdot 0.6065) \approx 2000/0.8787 \approx 2275$.

   Answer: Decreases; $y(10) \approx 2275$

3. A species of plant is introduced to an island. The population grows logistically. After 1 year, the population is 100. After 2 years, it is 400. If the carrying capacity is 1000, find the intrinsic growth rate r.

   - Use $y(t) = 1000/(1 + Ce^{-rt})$.
   - At $t = 1$: $100 = 1000/(1 + Ce^{-r}) \implies 1 + Ce^{-r} = 10 \implies Ce^{-r} = 9$.
   - At $t = 2$: $400 = 1000/(1 + Ce^{-2r}) \implies 1 + Ce^{-2r} = 2.5 \implies Ce^{-2r} = 1.5$.
   - Divide the two equations: $(Ce^{-r})/(Ce^{-2r}) = 9/1.5 \implies e^r = 6$.
   - Solve for r: $r = \ln(6) \approx 1.79$.

   Answer: $r = \ln(6) \approx 1.79$
