Newton's Law of Cooling

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Practice problems

1. A metal rod is placed in a room at $20°C$. The rod's initial temperature is $100°C$. After 1 hour, the temperature is $80°C$. Find the temperature of the rod after 2 hours.
2. A cake is removed from an oven at $180°C$ and placed in a room at $22°C$. After 10 minutes, the cake is $150°C$. How many minutes will it take for the cake to reach $100°C$?
3. An object is cooling in a room at T_s. Its temperature $T(t)$ is observed to follow $T(t) = T_s + 50e^{-0.05t}$. If the object's temperature is $40°C$ at $t = 0$ and the room is T_s, find T_s and the temperature after 20 minutes.

Solutions

1. A metal rod is placed in a room at $20°C$. The rod's initial temperature is $100°C$. After 1 hour, the temperature is $80°C$. Find the temperature of the rod after 2 hours.

   - Use the model $T(t) = 20 + (100 - 20)e^{-kt} = 20 + 80e^{-kt}$.
   - Use $T(1) = 80$ to find k: $80 = 20 + 80e^{-k} \implies 60 = 80e^{-k} \implies e^{-k} = 0.75$.
   - Find $k = -\ln(0.75) \approx 0.2877$.
   - Calculate $T(2) = 20 + 80(e^{-k})^2 = 20 + 80(0.75)^2$.
   - Compute $T(2) = 20 + 80(0.5625) = 20 + 45 = 65$.

   Answer: 65ˆ\circC

2. A cake is removed from an oven at $180°C$ and placed in a room at $22°C$. After 10 minutes, the cake is $150°C$. How many minutes will it take for the cake to reach $100°C$?

   - Model: $T(t) = 22 + (180 - 22)e^{-kt} = 22 + 158e^{-kt}$.
   - Find k using $T(10) = 150$: $150 = 22 + 158e^{-10k} \implies 128 = 158e^{-10k} \implies e^{-10k} = 128/158 \approx 0.8101$.
   - $-10k = \ln(0.8101) \implies k \approx 0.02106$.
   - Set $T(t) = 100$: $100 = 22 + 158e^{-0.02106t} \implies 78 = 158e^{-0.02106t}$.
   - Divide by 158: $e^{-0.02106t} \approx 0.4937$.
   - Take log: $-0.02106t = \ln(0.4937) \approx -0.7058 \implies t \approx 33.5$.

   Answer: \approx 33.5 \text minutes

3. An object is cooling in a room at T_s. Its temperature $T(t)$ is observed to follow $T(t) = T_s + 50e^{-0.05t}$. If the object's temperature is $40°C$ at $t = 0$ and the room is T_s, find T_s and the temperature after 20 minutes.

   - At $t = 0$, $T(0) = T_s + 50e^0 = T_s + 50$.
   - Given $T(0) = 40$, we have $40 = T_s + 50 \implies T_s = -10°C$.
   - The model is $T(t) = -10 + 50e^{-0.05t}$.
   - For $t = 20$: $T(20) = -10 + 50e^{-0.05(20)} = -10 + 50e^{-1}$.
   - Using $e^{-1} \approx 0.3679$, $T(20) = -10 + 50(0.3679) = -10 + 18.395 = 8.395$.

   Answer: T_s = -10ˆ\circC, T(20) \approx 8.4ˆ\circC