Exponential growth and decay; radiocarbon dating

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Practice problems

1. A sample of organic material is found to have 40% of the carbon-14 found in living organisms. Using a half-life of 5730 years, determine the age of the sample.
2. A population of insects grows at a rate proportional to its size. If the population doubles every 12 days, how long will it take for the population to triple in size?
3. A radioactive substance has a half-life of 150 years. If you start with 500 mg, how many years will it take for the substance to decay to 50 mg?

Solutions

1. A sample of organic material is found to have 40% of the carbon-14 found in living organisms. Using a half-life of 5730 years, determine the age of the sample.

   - Identify the ratio $y/y_0 = 0.40$.
   - Calculate the decay constant $k = \ln(0.5)/5730 \approx -0.00012097$.
   - Set up the equation $0.40 = e^{kt}$.
   - Solve for t by taking the natural log: $\ln(0.40) = kt$.
   - Compute $t = \ln(0.40)/(-0.00012097) \approx 7575$ years.

   Answer: 7575 \text years

2. A population of insects grows at a rate proportional to its size. If the population doubles every 12 days, how long will it take for the population to triple in size?

   - Use the doubling time to find k: $2y_0 = y_0e^{12k} \implies k = \ln(2)/12$.
   - Set up the equation for tripling: $3y_0 = y_0e^{kt}$.
   - Simplify to $3 = e^{(\ln(2)/12)t}$.
   - Take the natural log: $\ln(3) = (\ln(2)/12)t$.
   - Solve for $t = 12\ln(3)/\ln(2) \approx 19.02$ days.

   Answer: 19.02 \text days

3. A radioactive substance has a half-life of 150 years. If you start with 500 mg, how many years will it take for the substance to decay to 50 mg?

   - Find $k = \ln(0.5)/150 \approx -0.004621$.
   - Set up the equation $50 = 500e^{kt}$.
   - Divide by 500: $0.1 = e^{kt}$.
   - Take the natural log: $\ln(0.1) = kt$.
   - Solve for $t = \ln(0.1)/-0.004621 \approx 498.29$ years.

   Answer: 498.29 \text years
