Exact Equations

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Practice problems

1. Solve the exact equation $(3t^2 + 2ty)dt + t^2 dy = 0$.
2. Find the particular solution to $(2t + y)dt + (t + 2y)dy = 0$ passing through $(1, 1)$.
3. Solve the equation $\cos(t)e^y dt + \sin(t)e^y dy = 0$.

Solutions

1. Solve the exact equation $(3t^2 + 2ty)dt + t^2 dy = 0$.

   - Identify $M = 3t^2 + 2ty$ and $N = t^2$.
   - Check exactness: $\frac{\partial M}{\partial y} = 2t$ and $\frac{\partial N}{\partial t} = 2t$. It is exact.
   - Integrate M with respect to t: $\psi = \int (3t^2 + 2ty)dt = t^3 + t^2 y + g(y)$.
   - Differentiate ψ with respect to y: $\frac{\partial \psi}{\partial y} = t^2 + g'(y)$.
   - Set equal to N: $t^2 + g'(y) = t^2 \implies g'(y) = 0 \implies g(y) = C_1$.
   - The general solution is $t^3 + t^2 y = C$.

   Answer: tˆ3 + tˆ2 y = C

2. Find the particular solution to $(2t + y)dt + (t + 2y)dy = 0$ passing through $(1, 1)$.

   - Check exactness: $\frac{\partial M}{\partial y} = 1$ and $\frac{\partial N}{\partial t} = 1$. Exact.
   - Integrate M: $\psi = \int (2t + y)dt = t^2 + ty + g(y)$.
   - Differentiate ψ wrt y: $t + g'(y) = t + 2y \implies g'(y) = 2y \implies g(y) = y^2$.
   - General solution: $t^2 + ty + y^2 = C$.
   - Apply $(1, 1)$: $1^2 + (1)(1) + 1^2 = 3$, so $C = 3$.

   Answer: tˆ2 + ty + yˆ2 = 3

3. Solve the equation $\cos(t)e^y dt + \sin(t)e^y dy = 0$.

   - Check exactness: $\frac{\partial M}{\partial y} = \cos(t)e^y$ and $\frac{\partial N}{\partial t} = \cos(t)e^y$. Exact.
   - Integrate M: $\psi = \int \cos(t)e^y dt = \sin(t)e^y + g(y)$.
   - Differentiate ψ wrt y: $\sin(t)e^y + g'(y) = \sin(t)e^y \implies g'(y) = 0$.
   - General solution: $\sin(t)e^y = C$.

   Answer: \sin(t) eˆy = C
