

Separable Equations

Lesson 6 · Module 0 — What an ODE Is · Differential Equations · dailymaths.uk/diffeq

Practice problems

1. Find the general solution to the differential equation $y' = t^2y$.
2. Solve the initial value problem $y' = 2ty^2$ with $y(0) = 1$.
3. Find the general solution to $y' = y \cos(t)$. Identify any equilibrium solutions.

Solutions

1. Find the general solution to the differential equation $y' = t^2y$.

- Separate the variables: $\frac{1}{y}dy = t^2dt$.
- Integrate both sides: $\int \frac{1}{y}dy = \int t^2dt$.
- Compute integrals: $\ln|y| = \frac{1}{3}t^3 + C$.
- Exponentiate: $|y| = e^{\frac{1}{3}t^3+C} = e^C e^{\frac{1}{3}t^3}$.
- Simplify constant: $y(t) = Ae^{\frac{1}{3}t^3}$.

Answer: $y(t) = Ae^{\frac{1}{3}t^3}$

2. Solve the initial value problem $y' = 2ty^2$ with $y(0) = 1$.

- Separate variables: $\frac{1}{y^2}dy = 2tdt$.
- Integrate: $\int y^{-2}dy = \int 2tdt \implies -\frac{1}{y} = t^2 + C$.
- Apply initial condition $y(0) = 1$: $-\frac{1}{1} = 0^2 + C \implies C = -1$.
- Substitute C back: $-\frac{1}{y} = t^2 - 1$.
- Solve for y : $\frac{1}{y} = 1 - t^2 \implies y = \frac{1}{1-t^2}$.

Answer: $y(t) = \frac{1}{1-t^2}$

3. Find the general solution to $y' = y \cos(t)$. Identify any equilibrium solutions.

- Check for equilibrium: $h(y) = y = 0 \implies y(t) = 0$ is a solution.
- Separate variables: $\frac{1}{y}dy = \cos(t)dt$.
- Integrate: $\ln|y| = \sin(t) + C$.
- Exponentiate: $y = Ae^{\sin(t)}$.

Answer: $y(t) = Ae^{\sin(t)}$ and $y(t) = 0$