Verifying solutions; general vs particular; initial conditions

Practice problems

1. Verify that $y = 5e^{3t}$ is a solution to the differential equation $y' - 3y = 0$.
2. Given the general solution $y = Ce^{t/2}$, find the particular solution that satisfies the initial condition $y(0) = 10$.
3. Determine if $y = t^2 + 1$ is a solution to the ODE $y' = 2t$. If so, find the initial condition $y(0) = y_0$ that makes it a particular solution.

Solutions

1. Verify that $y = 5e^{3t}$ is a solution to the differential equation $y' - 3y = 0$.

   - Compute the derivative: $y' = 15e^{3t}$.
   - Substitute y and y' into the equation: $15e^{3t} - 3(5e^{3t})$.
   - Simplify: $15e^{3t} - 15e^{3t} = 0$.
   - The identity $0 = 0$ holds.

   Answer: It is a solution.

2. Given the general solution $y = Ce^{t/2}$, find the particular solution that satisfies the initial condition $y(0) = 10$.

   - Substitute $t = 0$ and $y = 10$ into the general solution: $10 = Ce^{0/2}$.
   - Since $e^0 = 1$, we have $10 = C(1)$.
   - Thus, $C = 10$.
   - Substitute C back into the general solution.

   Answer: $y = 10e^{t/2}$

3. Determine if $y = t^2 + 1$ is a solution to the ODE $y' = 2t$. If so, find the initial condition $y(0) = y_0$ that makes it a particular solution.

   - Compute the derivative: $y' = 2t$.
   - Substitute into the ODE: $2t = 2t$. This is an identity, so it is a solution.
   - To find the initial condition, evaluate y at $t = 0$: $y(0) = 0^2 + 1 = 1$.
   - The initial condition is $y(0) = 1$.

   Answer: Yes; $y(0) = 1$
