Order, linearity, solutions — the vocabulary

Lesson 2 · Module 0 — What an ODE Is · Differential Equations · dailymaths.uk/diffeq

Practice problems

1. Determine the order and linearity of the differential equation $y'' + y'y = \sin t$.
2. Verify if $y = e^{3t}$ is a solution to the ODE $y' - 3y = 0$.
3. Classify the ODE $y''' + t^2y'' - e^ty' + \cos(t)y = 0$ by order and linearity.

Solutions

1. Determine the order and linearity of the differential equation $y'' + y'y = \sin t$.

   - Identify the highest derivative: the second derivative y'' is present, so the order is 2.
   - Check for linearity: the term $y'y$ is a product of the unknown function and its derivative.
   - Since a product of y and its derivatives is present, the equation is non-linear.

   Answer: Order 2, Non-linear

2. Verify if $y = e^{3t}$ is a solution to the ODE $y' - 3y = 0$.

   - Compute the derivative of the candidate function: $y' = \frac{d}{dt}(e^{3t}) = 3e^{3t}$.
   - Substitute y and y' into the ODE: $3e^{3t} - 3(e^{3t})$.
   - Simplify the expression: $3e^{3t} - 3e^{3t} = 0$.
   - Since the result is $0 = 0$, the function is a solution.

   Answer: Yes, it is a solution

3. Classify the ODE $y''' + t^2y'' - e^t y' + \cos(t)y = 0$ by order and linearity.

   - Identify the highest derivative: y''' is the third derivative, so the order is 3.
   - Check for linearity: y''', y'', y', and y all appear to the first power.
   - Check coefficients: 1, t^2, $-e^t$, and $\cos(t)$ depend only on the independent variable t.
   - Since all conditions for linearity are met, the equation is linear.

   Answer: Order 3, Linear
