

Equations whose unknowns are functions

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Practice problems

1. Determine the order and whether the following equation is linear or non-linear: $y'' + 3y' + 2y = f(t)$.
2. Verify if the function $y(t) = e^{-5t}$ is a solution to the differential equation $y' + 5y = 0$.
3. Identify the order and linearity of the equation $y' = \frac{t}{y}$.

Solutions

1. Determine the order and whether the following equation is linear or non-linear: $y'' + 3y' + 2y = f(t)$.

- The highest derivative present is the second derivative y'' , so the order is 2.
- The function y and its derivatives y' and y'' all appear to the first power and are not multiplied by each other.
- The term $f(t)$ depends only on the independent variable t , which does not affect linearity.

Answer: Order 2, Linear

2. Verify if the function $y(t) = e^{-5t}$ is a solution to the differential equation $y' + 5y = 0$.

- First, compute the derivative of the proposed solution: $y' = -5e^{-5t}$.
- Substitute y and y' into the equation: $(-5e^{-5t}) + 5(e^{-5t})$.
- Simplify the expression: $-5e^{-5t} + 5e^{-5t} = 0$.
- Since the result is 0, the equation is satisfied.

Answer: Yes, it is a solution.

3. Identify the order and linearity of the equation $y' = \frac{t}{y}$.

- The highest derivative is y' , so the order is 1.
- Rewrite the equation as $yy' = t$.
- Because the unknown function y is multiplied by its derivative y' , the equation is non-linear.

Answer: Order 1, Non-linear