

Module 6 Review and Self-Test

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Practice problems

1. A sample of $n = 12$ measurements has a mean of $\bar{x} = 84$ and a sample standard deviation of $s = 6$. Compute a 99% confidence interval for the population mean.
2. In a survey of 500 people, 210 stated they prefer tea over coffee. Calculate a 95% confidence interval for the population proportion of tea lovers.
3. A pollster wants to estimate a population proportion with a margin of error of 2% at a 90% confidence level. What is the minimum sample size required if no prior estimate of the proportion is available?

Solutions

1. A sample of $n = 12$ measurements has a mean of $\bar{x} = 84$ and a sample standard deviation of $s = 6$. Compute a 99% confidence interval for the population mean.

- Identify that σ is unknown, so use the t-distribution with $df = 12 - 1 = 11$.
- Find the critical value t^* for 99% confidence and 11 degrees of freedom, which is approximately 3.106.
- Calculate the standard error: $SE = 6/\sqrt{12} \approx 1.732$.
- Calculate the margin of error: $MOE = 3.106 \times 1.732 \approx 5.38$.
- The interval is 84 ± 5.38 .

Answer: [78.62, 89.38]

2. In a survey of 500 people, 210 stated they prefer tea over coffee. Calculate a 95% confidence interval for the population proportion of tea lovers.

- Calculate the sample proportion: $\hat{p} = 210/500 = 0.42$.
- Identify the critical value z^* for 95% confidence, which is 1.96.
- Calculate the standard error: $SE = \sqrt{(0.42 \times 0.58)/500} \approx 0.0220$.
- Calculate the margin of error: $MOE = 1.96 \times 0.0220 \approx 0.043$.
- The interval is 0.42 ± 0.043 .

Answer: [0.377, 0.463]

3. A pollster wants to estimate a population proportion with a margin of error of 2% at a 90% confidence level. What is the minimum sample size required if no prior estimate of the proportion is available?

- Use the conservative estimate $\hat{p} = 0.5$.
- Identify the critical value z^* for 90% confidence, which is 1.645.
- Set the margin of error $MOE = 0.02$.
- Apply the formula: $n = (1.645/0.02)^2 \times 0.5 \times 0.5$.
- Calculate: $n = (82.25)^2 \times 0.25 = 6765.0625 \times 0.25 = 1691.26$.
- Round up to the nearest whole number.

Answer: 1692