

Sample size planning

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Practice problems

1. Find the minimum sample size n required to estimate a population proportion with a margin of error of 0.04 at a 95% confidence level, assuming no prior knowledge of the proportion.
2. A researcher believes the proportion of a certain trait in a population is approximately 30%. How many people must be sampled to be 90% confident that the estimate is within 3% of the true proportion?
3. You want to reduce your margin of error from 5% to 2.5% while keeping the confidence level and the proportion estimate the same. By what factor must the sample size n increase?

Solutions

1. Find the minimum sample size n required to estimate a population proportion with a margin of error of 0.04 at a 95% confidence level, assuming no prior knowledge of the proportion.

- Identify the critical value for 95% confidence: $z^* = 1.96$.
- Use the conservative estimate $\hat{p} = 0.5$, so $\hat{p}(1 - \hat{p}) = 0.25$.
- Plug values into the formula: $n = \frac{0.25(1.96)^2}{(0.04)^2}$.
- Calculate: $n = \frac{0.9604}{0.0016} = 600.25$.
- Round up to the nearest integer: $n = 601$.

Answer: 601

2. A researcher believes the proportion of a certain trait in a population is approximately 30%. How many people must be sampled to be 90% confident that the estimate is within 3% of the true proportion?

- Identify the critical value for 90% confidence: $z^* = 1.645$.
- Use the prior estimate $\hat{p} = 0.3$, so $\hat{p}(1 - \hat{p}) = 0.3 \times 0.7 = 0.21$.
- Set the margin of error $\text{MOE} = 0.03$.
- Plug into the formula: $n = \frac{(1.645)^2(0.21)}{(0.03)^2}$.
- Calculate: $n = \frac{2.706025 \times 0.21}{0.0009} = \frac{0.56826525}{0.0009} \approx 631.41$.
- Round up to the nearest integer: $n = 632$.

Answer: 632

3. You want to reduce your margin of error from 5% to 2.5% while keeping the confidence level and the proportion estimate the same. By what factor must the sample size n increase?

- The formula for n is $n = \frac{C}{\text{MOE}^2}$ where C is a constant for a fixed z^* and $\hat{p}$.
- Let n_1 be the size for $\text{MOE}_1 = 0.05$ and n_2 be the size for $\text{MOE}_2 = 0.025$.
- The ratio is $\frac{n_2}{n_1} = \frac{C/(0.025)^2}{C/(0.05)^2} = \frac{(0.05)^2}{(0.025)^2}$.
- Simplify the ratio: $\left(\frac{0.05}{0.025}\right)^2 = 2^2 = 4$.
- The sample size must increase by a factor of 4.

Answer: 4