

The t-distribution; intervals with sigma unknown

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Practice problems

1. A sample of $n = 15$ measurements has a mean of $\bar{x} = 82$ and a sample standard deviation of $s = 6$. Compute a 95% confidence interval for the population mean. Use $t^*(df = 14, 95\%) = 2.145$.
2. A researcher finds that $n = 20$ samples of a chemical have an average purity of 98.5% with a sample standard deviation of 0.8%. Find the 99% confidence interval. Use $t^*(df = 19, 99\%) = 2.861$.
3. You have a sample of $n = 5$ items with $\bar{x} = 10$ and $s = 2$. You want the margin of error to be no more than 1.5. Is a 90% confidence interval sufficient? Use $t^*(df = 4, 90\%) = 2.132$.

Solutions

1. A sample of $n = 15$ measurements has a mean of $\bar{x} = 82$ and a sample standard deviation of $s = 6$. Compute a 95% confidence interval for the population mean. Use $t^*(df = 14, 95\%) = 2.145$.

- Identify the given values: $\bar{x} = 82$, $s = 6$, $n = 15$.
- Calculate the standard error: $SE = 6/\sqrt{15} \approx 1.549$.
- Calculate the margin of error: $ME = 2.145 \times 1.549 \approx 3.323$.
- Construct the interval: 82 ± 3.323 .

Answer: [78.677, 85.323]

2. A researcher finds that $n = 20$ samples of a chemical have an average purity of 98.5% with a sample standard deviation of 0.8%. Find the 99% confidence interval. Use $t^*(df = 19, 99\%) = 2.861$.

- Identify the given values: $\bar{x} = 98.5$, $s = 0.8$, $n = 20$.
- Calculate the standard error: $SE = 0.8/\sqrt{20} \approx 0.1789$.
- Calculate the margin of error: $ME = 2.861 \times 0.1789 \approx 0.512$.
- Construct the interval: 98.5 ± 0.512 .

Answer: [97.988, 99.012]

3. You have a sample of $n = 5$ items with $\bar{x} = 10$ and $s = 2$. You want the margin of error to be no more than 1.5. Is a 90% confidence interval sufficient? Use $t^*(df = 4, 90\%) = 2.132$.

- Calculate the standard error: $SE = 2/\sqrt{5} \approx 0.8944$.
- Calculate the margin of error for 90% confidence: $ME = 2.132 \times 0.8944 \approx 1.907$.
- Compare the calculated margin of error 1.907 to the required 1.5.
- Since $1.907 > 1.5$, the interval is too wide.

Answer: No, the margin of error is 1.907, which exceeds 1.5.