

Confidence intervals II — for a mean, sigma known

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Practice problems

1. A researcher finds that a sample of $n = 36$ lightbulbs has a mean life of $\bar{x} = 72$ hours. The population standard deviation is known to be $\sigma = 6$ hours. Calculate a 99% confidence interval for the true population mean.
2. A company knows the population standard deviation of the weight of its cereal boxes is $\sigma = 0.5$ ounces. A sample of $n = 64$ boxes has a mean weight of $\bar{x} = 16.2$ ounces. Find the 95% confidence interval for the population mean.
3. You want to estimate the average height of a plant species with 95% confidence. You know $\sigma = 3$ cm. If you want the margin of error to be no more than 0.5 cm, what is the minimum sample size n required?

Solutions

1. A researcher finds that a sample of $n = 36$ lightbulbs has a mean life of $\bar{x} = 72$ hours. The population standard deviation is known to be $\sigma = 6$ hours. Calculate a 99% confidence interval for the true population mean.

- Identify the knowns: $\bar{x} = 72$, $\sigma = 6$, $n = 36$.
- For 99% confidence, the critical value z is 2.58.
- Calculate the standard error: $SE = 6/\sqrt{36} = 6/6 = 1$.
- Calculate the margin of error: $ME = 2.58 \times 1 = 2.58$.
- Construct the interval: 72 ± 2.58 .

Answer: [69.42, 74.58]

2. A company knows the population standard deviation of the weight of its cereal boxes is $\sigma = 0.5$ ounces. A sample of $n = 64$ boxes has a mean weight of $\bar{x} = 16.2$ ounces. Find the 95% confidence interval for the population mean.

- Identify the knowns: $\bar{x} = 16.2$, $\sigma = 0.5$, $n = 64$.
- For 95% confidence, $z = 1.96$.
- Calculate the standard error: $SE = 0.5/\sqrt{64} = 0.5/8 = 0.0625$.
- Calculate the margin of error: $ME = 1.96 \times 0.0625 = 0.1225$.
- Construct the interval: 16.2 ± 0.1225 .

Answer: [16.0775, 16.3225]

3. You want to estimate the average height of a plant species with 95% confidence. You know $\sigma = 3$ cm. If you want the margin of error to be no more than 0.5 cm, what is the minimum sample size n required?

- Set up the margin of error formula: $ME = z \frac{\sigma}{\sqrt{n}}$.
- Substitute the knowns: $0.5 = 1.96 \times \frac{3}{\sqrt{n}}$.
- Solve for $\sqrt{n}$: $\sqrt{n} = (1.96 \times 3)/0.5 = 5.88/0.5 = 11.76$.
- Square both sides: $n = 11.76^2 = 138.2976$.
- Since n must be an integer, round up to the nearest whole number.

Answer: 139