

Confidence intervals I — the idea

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Practice problems

1. A researcher calculates a 95% confidence interval for the average weight of a fruit to be [140g, 160g]. Which of the following is the correct interpretation? A) There is a 95% probability that the true mean weight is between 140g and 160g. B) 95% of all fruits in the population weigh between 140g and 160g. C) If we repeated this sampling process many times, 95% of the resulting intervals would contain the true mean weight.
2. Suppose you have a 95% confidence interval for a population mean. If you decide to increase the confidence level to 99% while keeping the sample size and the data the same, what happens to the width of the interval?
3. A scientist finds that a sample of size $n = 25$ gives a 95% confidence interval width of 10 units. If the scientist increases the sample size to $n = 100$ and finds the same sample mean and standard deviation, what is the new width of the 95% confidence interval?

Solutions

1. A researcher calculates a 95% confidence interval for the average weight of a fruit to be [140g, 160g]. Which of the following is the correct interpretation? A) There is a 95% probability that the true mean weight is between 140g and 160g. B) 95% of all fruits in the population weigh between 140g and 160g. C) If we repeated this sampling process many times, 95% of the resulting intervals would contain the true mean weight.
 - Option A is incorrect because the true mean is a fixed constant, not a random variable; it is either in the interval or it is not.
 - Option B is incorrect because the confidence interval estimates the population mean, not the distribution of individual values.
 - Option C correctly describes the confidence level as the long-run success rate of the interval-generating procedure.

Answer: C

2. Suppose you have a 95% confidence interval for a population mean. If you decide to increase the confidence level to 99% while keeping the sample size and the data the same, what happens to the width of the interval?
 - To be more confident that the interval captures the true mean, we must account for more of the sampling distribution.
 - This requires a larger multiplier for the margin of error.
 - A larger multiplier increases the distance from the point estimate to the boundaries.

Answer: The interval becomes wider.

3. A scientist finds that a sample of size $n = 25$ gives a 95% confidence interval width of 10 units. If the scientist increases the sample size to $n = 100$ and finds the same sample mean and standard deviation, what is the new width of the 95% confidence interval?
 - The width of the interval is proportional to the standard error, which is $\frac{\sigma}{\sqrt{n}}$.
 - The original width is proportional to $1/\sqrt{25} = 1/5$.
 - The new width is proportional to $1/\sqrt{100} = 1/10$.
 - The ratio of the new width to the old width is $(1/10)/(1/5) = 1/2$.
 - Therefore, the new width is $10 \times 0.5 = 5$.

Answer: 5 units