

Maximum likelihood II — worked derivations

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Practice problems

1. A basketball player makes 12 out of 15 free throws. Using the Bernoulli MLE derivation, find the estimate $\hat{p}$ for the probability of success.
2. A call center receives an average of $x_1 = 5, x_2 = 8, x_3 = 2$ calls per minute over three minutes. Find the MLE $\hat{\lambda}$ for the Poisson rate of calls per minute.
3. Suppose you have a sample of n observations from a Normal distribution with known variance σ^2 . Show that the log-likelihood $l(\mu)$ is maximized when μ equals the sample mean $\bar{X}$.

Solutions

1. A basketball player makes 12 out of 15 free throws. Using the Bernoulli MLE derivation, find the estimate $\hat{p}$ for the probability of success.

- Identify the number of trials $n = 15$ and the number of successes $\sum x_i = 12$.
- Use the derived Bernoulli MLE formula $\hat{p} = \frac{\sum x_i}{n}$.
- Calculate $\hat{p} = \frac{12}{15}$.
- Simplify the fraction to 0.8.

Answer: 0.8

2. A call center receives an average of $x_1 = 5, x_2 = 8, x_3 = 2$ calls per minute over three minutes. Find the MLE $\hat{\lambda}$ for the Poisson rate of calls per minute.

- Identify the sample data: 5, 8, 2 with $n = 3$.
- Use the Poisson MLE formula $\hat{\lambda} = \frac{\sum x_i}{n}$.
- Sum the observations: $5 + 8 + 2 = 15$.
- Divide by the sample size: $15/3 = 5$.

Answer: 5

3. Suppose you have a sample of n observations from a Normal distribution with known variance σ^2 . Show that the log-likelihood $l(\mu)$ is maximized when μ equals the sample mean $\bar{X}$.

- Write the log-likelihood: $l(\mu) = -\frac{n}{2} \ln(2\pi\sigma^2) - \frac{1}{2\sigma^2} \sum (x_i - \mu)^2$.
- Differentiate with respect to μ : $\frac{d}{d\mu} l(\mu) = \frac{1}{\sigma^2} \sum (\mu - x_i)$.
- Set the derivative to zero: $\sum (\mu - x_i) = 0$.
- Expand the sum: $n\mu - \sum x_i = 0$.
- Solve for μ : $\mu = \frac{\sum x_i}{n} = \bar{X}$.

Answer: $\hat{\mu} = \bar{X}$