

Maximum likelihood I — the principle

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Practice problems

1. A coin is flipped 4 times, resulting in 3 heads and 1 tail. Find the Maximum Likelihood Estimate for the probability of heads θ .
2. For an exponential distribution with density $f(x|\lambda) = \lambda e^{-\lambda x}$, you observe a single data point $x = 5$. Find the MLE for λ .
3. Suppose you have a Poisson distribution and observe two samples: $x_1 = 2$ and $x_2 = 4$. Find the MLE for λ .

Solutions

1. A coin is flipped 4 times, resulting in 3 heads and 1 tail. Find the Maximum Likelihood Estimate for the probability of heads θ .

- The likelihood is $\mathcal{L}(\theta) = \theta^3(1 - \theta)^1$.
- The log-likelihood is $l(\theta) = 3 \ln(\theta) + \ln(1 - \theta)$.
- The derivative is $l'(\theta) = \frac{3}{\theta} - \frac{1}{1-\theta}$.
- Setting $l'(\theta) = 0$ gives $3(1 - \theta) = \theta$, so $3 - 3\theta = \theta$.
- Solving for θ gives $4\theta = 3$, so $\theta = 0.75$.

Answer: $\hat{\theta}_{MLE} = 0.75$

2. For an exponential distribution with density $f(x|\lambda) = \lambda e^{-\lambda x}$, you observe a single data point $x = 5$. Find the MLE for λ .

- The likelihood is $\mathcal{L}(\lambda) = \lambda e^{-5\lambda}$.
- The log-likelihood is $l(\lambda) = \ln(\lambda) - 5\lambda$.
- The derivative is $l'(\lambda) = \frac{1}{\lambda} - 5$.
- Setting $l'(\lambda) = 0$ gives $\frac{1}{\lambda} = 5$.
- Solving for λ gives $\lambda = 0.2$.

Answer: $\hat{\lambda}_{MLE} = 0.2$

3. Suppose you have a Poisson distribution and observe two samples: $x_1 = 2$ and $x_2 = 4$. Find the MLE for λ .

- The likelihood is $\mathcal{L}(\lambda) = \frac{e^{-\lambda}\lambda^2}{2!} \cdot \frac{e^{-\lambda}\lambda^4}{4!}$.
- The log-likelihood is $l(\lambda) = -2\lambda + 2 \ln(\lambda) - \ln(2!) - 4\lambda + 4 \ln(\lambda) - \ln(4!)$.
- The derivative is $l'(\lambda) = -2 + \frac{2}{\lambda} - 4 + \frac{4}{\lambda}$.
- Setting $l'(\lambda) = 0$ gives $\frac{6}{\lambda} = 2$.
- Solving for λ gives $\lambda = 3$.

Answer: $\hat{\lambda}_{MLE} = 3$