

Point estimation; bias and variance of estimators

Lesson 79 · Module 6 — Estimation · Probability & Statistics · dailymaths.uk/probstat

Practice problems

1. Consider a sample X_1, X_2 from a population with mean μ . We propose the estimator $\hat{\theta} = \frac{X_1 + 2X_2}{3}$. Is this estimator unbiased for μ ?
2. An estimator $\hat{\theta}$ has a variance of 0.25 and a bias of 0.5. Calculate the Mean Squared Error (MSE) of this estimator.
3. Suppose we estimate the population mean μ using $\hat{\theta} = \bar{X} + \frac{1}{n}$, where $\bar{X}$ is the sample mean and n is the sample size. Find the bias of this estimator and determine if it is unbiased as n approaches infinity.

Solutions

1. Consider a sample X_1, X_2 from a population with mean μ . We propose the estimator $\hat{\theta} = \frac{X_1 + 2X_2}{3}$. Is this estimator unbiased for μ ?

- Find the expected value: $E[\hat{\theta}] = E[\frac{X_1 + 2X_2}{3}]$
- Use linearity of expectation: $E[\hat{\theta}] = \frac{1}{3}(E[X_1] + 2E[X_2])$
- Substitute $E[X_1] = \mu$ and $E[X_2] = \mu$: $E[\hat{\theta}] = \frac{1}{3}(\mu + 2\mu) = \frac{3\mu}{3} = \mu$
- Since $E[\hat{\theta}] = \mu$, the bias is zero.

Answer: Yes, it is unbiased.

2. An estimator $\hat{\theta}$ has a variance of 0.25 and a bias of 0.5. Calculate the Mean Squared Error (MSE) of this estimator.

- Identify the given values: $\text{Var}(\hat{\theta}) = 0.25$ and $\text{Bias}(\hat{\theta}) = 0.5$
- Use the MSE formula: $\text{MSE} = \text{Var}(\hat{\theta}) + [\text{Bias}(\hat{\theta})]^2$
- Substitute the values: $\text{MSE} = 0.25 + (0.5)^2$
- Compute the square: $0.5^2 = 0.25$
- Sum the results: $0.25 + 0.25 = 0.5$

Answer: 0.5

3. Suppose we estimate the population mean μ using $\hat{\theta} = \bar{X} + \frac{1}{n}$, where $\bar{X}$ is the sample mean and n is the sample size. Find the bias of this estimator and determine if it is unbiased as n approaches infinity.

- Find the expectation: $E[\hat{\theta}] = E[\bar{X} + \frac{1}{n}] = E[\bar{X}] + \frac{1}{n}$
- Since $E[\bar{X}] = \mu$, we have $E[\hat{\theta}] = \mu + \frac{1}{n}$
- Calculate bias: $\text{Bias} = E[\hat{\theta}] - \mu = (\mu + \frac{1}{n}) - \mu = \frac{1}{n}$
- Take the limit as $n \rightarrow \infty$: $\lim_{n \rightarrow \infty} \frac{1}{n} = 0$

Answer: The bias is $1/n$; it becomes unbiased in the limit as n approaches infinity.