

Populations, samples, and sampling distributions

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Practice problems

1. A population has a standard deviation of $\sigma = 24$. If a random sample of size $n = 64$ is taken, what is the standard error of the sample mean?
2. A population has a mean $\mu = 50$ and a standard deviation $\sigma = 10$. For a sample of size $n = 25$, what is the probability that the sample mean $\bar{X}$ is less than 48?
3. You want to estimate a population mean with a standard deviation of $\sigma = 12$. You want the standard error of your sample mean to be no more than 1.5. What is the minimum sample size n required?

Solutions

1. A population has a standard deviation of $\sigma = 24$. If a random sample of size $n = 64$ is taken, what is the standard error of the sample mean?

- Identify the population standard deviation $\sigma = 24$ and sample size $n = 64$.
- Use the formula $SE = \frac{\sigma}{\sqrt{n}}$.
- Calculate the square root of 64, which is 8.
- Divide 24 by 8 to get 3.

Answer: 3

2. A population has a mean $\mu = 50$ and a standard deviation $\sigma = 10$. For a sample of size $n = 25$, what is the probability that the sample mean $\bar{X}$ is less than 48?

- Calculate the standard error: $SE = \frac{10}{\sqrt{25}} = \frac{10}{5} = 2$.
- Calculate the z-score for $\bar{X} = 48$: $z = \frac{48-50}{2} = \frac{-2}{2} = -1$.
- Find the probability $P(Z < -1)$ using the standard normal distribution.
- The area to the left of $z = -1$ is approximately 0.1587.

Answer: 0.1587

3. You want to estimate a population mean with a standard deviation of $\sigma = 12$. You want the standard error of your sample mean to be no more than 1.5. What is the minimum sample size n required?

- Set up the inequality $SE = \frac{12}{\sqrt{n}} \leq 1.5$.
- Rearrange to solve for $\sqrt{n}$: $\sqrt{n} \geq \frac{12}{1.5}$.
- Calculate $12/1.5 = 8$.
- Square both sides to find n : $n \geq 8^2 = 64$.

Answer: 64