

From probability to statistics: the inference U-turn

Lesson 77 · Module 6 — Estimation · Probability & Statistics · dailymaths.uk/probstat

Practice problems

1. A researcher knows that a certain drug has a 20% success rate. They want to find the probability that at least 3 out of 10 patients recover. Is this a forward probability problem or an inverse inference problem? Explain why.
2. A quality control engineer tests 50 components and finds that 2 are defective. They want to estimate the overall defect rate of the entire production line. Is this a forward probability problem or an inverse inference problem? Explain why.
3. Consider a scenario where you are told that a population follows a normal distribution with $\mu = 100$ and $\sigma = 15$. You are asked to find the percentage of the population with a score above 130. Then, you are told you have a sample of 30 people with an average score of 105, and you are asked to guess the true population mean. Contrast these two tasks in terms of the 'inference U-turn'.

Solutions

1. A researcher knows that a certain drug has a 20% success rate. They want to find the probability that at least 3 out of 10 patients recover. Is this a forward probability problem or an inverse inference problem? Explain why.

- Identify the knowns: The success rate $p = 0.2$ is given.
- Identify the goal: Find the probability of a specific data outcome (at least 3 recoveries).
- Since we move from a known parameter to a predicted outcome, it is a forward problem.

Answer: Forward probability problem.

2. A quality control engineer tests 50 components and finds that 2 are defective. They want to estimate the overall defect rate of the entire production line. Is this a forward probability problem or an inverse inference problem? Explain why.

- Identify the knowns: The data is fixed (2 defects out of 50).
- Identify the goal: Estimate the population parameter p .
- Since we move from observed data to an unknown parameter, it is an inverse inference problem.

Answer: Inverse inference problem.

3. Consider a scenario where you are told that a population follows a normal distribution with $\mu = 100$ and $\sigma = 15$. You are asked to find the percentage of the population with a score above 130. Then, you are told you have a sample of 30 people with an average score of 105, and you are asked to guess the true population mean. Contrast these two tasks in terms of the 'inference U-turn'.

- Task 1: Parameters $\mu = 100, \sigma = 15$ are known. We predict the data (percentage above 130). This is the forward direction.
- Task 2: Data (sample mean 105) is known. We estimate the parameter μ . This is the inverse direction.
- The shift from Task 1 to Task 2 represents the U-turn from probability to statistics.

Answer: Task 1 is forward (probability); Task 2 is inverse (inference).