

Module 5 review + self-test

Lesson 76 · Module 5 — Limit Theorems · Probability & Statistics · dailymaths.uk/probstat

Practice problems

1. A sample of $n = 50$ independent observations is taken from a distribution with mean $\mu = 10$ and variance $\sigma^2 = 4$. Use the Central Limit Theorem to estimate the probability that the sample mean $\bar{X}$ is greater than 10.5.
2. A random variable X has a mean of 100 and a standard deviation of 15. Without knowing the distribution of X , what is the maximum possible probability that X is less than 55 or greater than 145?
3. A particle performs a 1D random walk starting at 0. In each step, it moves +1 with probability 0.7 and -1 with probability 0.3. After $n = 100$ steps, what is the probability that the particle is at position $S_{100} \geq 50$? Use the normal approximation with continuity correction.

Solutions

1. A sample of $n = 50$ independent observations is taken from a distribution with mean $\mu = 10$ and variance $\sigma^2 = 4$. Use the Central Limit Theorem to estimate the probability that the sample mean $\bar{X}$ is greater than 10.5.

- Identify the parameters: $\mu = 10$, $\sigma = 2$, $n = 50$.
- Calculate the standard error: $\sigma_{\bar{X}} = \frac{2}{\sqrt{50}} \approx 0.2828$.
- Calculate the z-score: $z = \frac{10.5-10}{0.2828} \approx 1.768$.
- Find the probability $P(Z > 1.768)$ using the normal table, which is approximately 0.0385.

Answer: 0.0385

2. A random variable X has a mean of 100 and a standard deviation of 15. Without knowing the distribution of X , what is the maximum possible probability that X is less than 55 or greater than 145?

- The range is 100 ± 45 .
- Calculate k as the number of standard deviations: $k = \frac{45}{15} = 3$.
- Apply Chebyshev's inequality: $P(|X - 100| \geq 45) \leq \frac{1}{3^2}$.
- Compute the result: $\frac{1}{9} \approx 0.1111$.

Answer: 0.1111

3. A particle performs a 1D random walk starting at 0. In each step, it moves +1 with probability 0.7 and -1 with probability 0.3. After $n = 100$ steps, what is the probability that the particle is at position $S_{100} \geq 50$? Use the normal approximation with continuity correction.

- Expected value of one step: $E[X] = 0.7 - 0.3 = 0.4$.
- Variance of one step: $\text{Var}(X) = 1 - (0.4)^2 = 0.84$.
- For $n = 100$: $\mu_S = 100(0.4) = 40$ and $\sigma_S = \sqrt{100(0.84)} = \sqrt{84} \approx 9.165$.
- Apply continuity correction: $P(S_{100} \geq 50) \approx P(S_{norm} \geq 49.5)$.
- Calculate z-score: $z = \frac{49.5-40}{9.165} \approx 1.036$.
- Find $P(Z > 1.036) \approx 0.1501$.

Answer: 0.1501