

Random walks and the drunkard's return

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Practice problems

1. A random walker in one dimension starts at 0. After 12 steps, what is the probability that the walker is at position 4?
2. In a two dimensional random walk, a walker takes 2 steps. What is the probability they return to the origin $(0, 0)$?
3. A one dimensional walker takes 100 steps. Using the normal approximation, what is the probability that the walker ends up at a position $S_{100} > 20$?

Solutions

1. A random walker in one dimension starts at 0. After 12 steps, what is the probability that the walker is at position 4?

- Let r be right steps and l be left steps. We need $r - l = 4$ and $r + l = 12$.
- Adding the equations: $2r = 16$, so $r = 8$. Then $l = 4$.
- The probability is $\binom{12}{8}(0.5)^{12}$.
- $\binom{12}{8} = 495$.
- $495/4096 \approx 0.1208$.

Answer: 0.1208

2. In a two dimensional random walk, a walker takes 2 steps. What is the probability they return to the origin $(0, 0)$?

- Total paths are $4^2 = 16$.
- To return, the walker must move in opposite directions: (Right, Left), (Left, Right), (Up, Down), or (Down, Up).
- There are 4 such paths.
- Probability is $4/16 = 0.25$.

Answer: 0.25

3. A one dimensional walker takes 100 steps. Using the normal approximation, what is the probability that the walker ends up at a position $S_{100} > 20$?

- The mean is $E[S_{100}] = 0$.
- The standard deviation is $\sigma = \sqrt{100} = 10$.
- We calculate the z-score: $Z = (20 - 0)/10 = 2$.
- We need $P(Z > 2)$.
- Using the standard normal table, $P(Z \leq 2) \approx 0.9772$.
- The probability is $1 - 0.9772 = 0.0228$.

Answer: 0.0228