Simulation as a tool: Monte Carlo methods

Practice problems

1. In a Buffon's needle experiment, the needle length L is exactly half the line spacing d. In a simulation of 500 drops, 112 needles cross a line. Use this result to estimate the value of π.

2. We want to estimate the area of the region where $y \leq x^2$ within the unit square ($0 \leq x \leq 1, 0 \leq y \leq 1$). If 1000 random points are generated and 330 fall within the region, what is the estimated area?

3. A region is defined by $x^2 + y^2 \leq 1$ and $y \geq 0$. We enclose this in a box where $-1 \leq x \leq 1$ and $0 \leq y \leq 1$. If we throw 2000 darts and 1580 land inside the region, estimate the value of π.

Solutions

1. In a Buffon's needle experiment, the needle length L is exactly half the line spacing d. In a simulation of 500 drops, 112 needles cross a line. Use this result to estimate the value of π.

   - The theoretical probability is $P = \frac{2L}{\pi d}$. Substituting $L = d/2$, we get $P = \frac{2(d/2)}{\pi d} = \frac{1}{\pi}$.
   - The observed fraction is $\frac{112}{500} = 0.224$.
   - Setting the theoretical probability equal to the observed fraction: $\frac{1}{\pi} \approx 0.224$.
   - Solving for π, we get $\pi \approx \frac{1}{0.224} \approx 4.464$.

   Answer: 4.464

2. We want to estimate the area of the region where $y \leq x^2$ within the unit square ($0 \leq x \leq 1, 0 \leq y \leq 1$). If 1000 random points are generated and 330 fall within the region, what is the estimated area?

   - The area of the bounding box (the unit square) is $1 \times 1 = 1$.
   - The fraction of points falling inside the region is $\frac{330}{1000} = 0.33$.
   - The estimated area is $\text{Area}_{\text{box}} \times \text{fraction} = 1 \times 0.33 = 0.33$.

   Answer: 0.33

3. A region is defined by $x^2 + y^2 \leq 1$ and $y \geq 0$. We enclose this in a box where $-1 \leq x \leq 1$ and $0 \leq y \leq 1$. If we throw 2000 darts and 1580 land inside the region, estimate the value of π.

   - The area of the box is $2 \times 1 = 2$.
   - The estimated area of the region is $2 \times \frac{1580}{2000} = 2 \times 0.79 = 1.58$.
   - The region is a semi-circle of radius 1, so its exact area is $\frac{1}{2}\pi(1)^2 = \frac{\pi}{2}$.
   - Setting $\frac{\pi}{2} \approx 1.58$, we find $\pi \approx 1.58 \times 2 = 3.16$.

   Answer: 3.16