

CLT in action: normal approximation to the binomial

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Practice problems

1. A biased coin has a probability of heads $p = 0.6$. If the coin is flipped $n = 80$ times, use the normal approximation with continuity correction to find the probability of getting 45 or fewer heads.
2. A factory produces lightbulbs with a 5% defect rate. In a sample of 200 bulbs, what is the probability that more than 12 are defective? Use the normal approximation with continuity correction.
3. A multiple-choice test has 50 questions, each with 4 options. If a student guesses randomly on every question, what is the probability they get between 10 and 15 questions correct, inclusive? Use the normal approximation with continuity correction.

Solutions

1. A biased coin has a probability of heads $p = 0.6$. If the coin is flipped $n = 80$ times, use the normal approximation with continuity correction to find the probability of getting 45 or fewer heads.

- Check conditions: $np = 80 \times 0.6 = 48$ and $nq = 80 \times 0.4 = 32$. Both are ≥ 5 .
- Parameters: $\mu = 48$, $\sigma = \sqrt{80 \times 0.6 \times 0.4} = \sqrt{19.2} \approx 4.38$.
- Continuity correction: $P(X \leq 45) \approx P(Y < 45.5)$.
- Z-score: $Z = \frac{45.5-48}{4.38} = \frac{-2.5}{4.38} \approx -0.57$.
- Probability: $P(Z < -0.57) \approx 0.2843$.

Answer: 0.2843

2. A factory produces lightbulbs with a 5% defect rate. In a sample of 200 bulbs, what is the probability that more than 12 are defective? Use the normal approximation with continuity correction.

- Check conditions: $np = 200 \times 0.05 = 10$ and $nq = 200 \times 0.95 = 190$. Both are ≥ 5 .
- Parameters: $\mu = 10$, $\sigma = \sqrt{200 \times 0.05 \times 0.95} = \sqrt{9.5} \approx 3.08$.
- Continuity correction: $P(X > 12) \approx P(Y > 12.5)$.
- Z-score: $Z = \frac{12.5-10}{3.08} = \frac{2.5}{3.08} \approx 0.81$.
- Probability: $P(Z > 0.81) = 1 - 0.7910 = 0.2090$.

Answer: 0.2090

3. A multiple-choice test has 50 questions, each with 4 options. If a student guesses randomly on every question, what is the probability they get between 10 and 15 questions correct, inclusive? Use the normal approximation with continuity correction.

- Check conditions: $np = 50 \times 0.25 = 12.5$ and $nq = 50 \times 0.75 = 37.5$. Both are ≥ 5 .
- Parameters: $\mu = 12.5$, $\sigma = \sqrt{50 \times 0.25 \times 0.75} = \sqrt{9.375} \approx 3.06$.
- Continuity correction: $P(10 \leq X \leq 15) \approx P(9.5 < Y < 15.5)$.
- Z-scores: $Z_1 = \frac{9.5-12.5}{3.06} \approx -0.98$ and $Z_2 = \frac{15.5-12.5}{3.06} \approx 0.98$.
- Probability: $P(-0.98 < Z < 0.98) = 0.8365 - 0.1635 = 0.6730$.

Answer: 0.6730